Semester-1

Unit-1

Paper - CC-2

MDR

Theory of equations: Relation between roots and coefficients

Let us consider a general equation of nth degree

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n = 0 \quad \text{(1)}$$

where $a_0 \neq 0$.

Suppose $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ are the n roots of equation (1).

Then

$$a_0(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)\cdots(x-\alpha_n) = 0 \quad \text{(2)}$$

Comparing equation (1) and (2) we get the relation between roots and coefficients as follows.

$$\alpha_1+\alpha_2+\alpha_3+\cdots+\alpha_n = \sum\alpha_1 = -\frac{a_1}{a_0}$$

$$\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_4+\cdots+\alpha_{n-1}\alpha_n+\alpha_n\alpha_1 = \sum\alpha_1\alpha_2 = \frac{a_2}{a_0}$$

$$\alpha_1\alpha_2\alpha_3+\alpha_2\alpha_3\alpha_4+\alpha_3\alpha_4\alpha_5+\cdots+\alpha_n\alpha_1\alpha_2 = \sum\alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0}$$

$$\vdots$$

$$\alpha_1\alpha_2\alpha_3\cdots\alpha_n = \prod\alpha_1 = (-1)^n\frac{a_n}{a_0}$$

For example

(a) Let α, β be the roots of equation $a_0x^2+a_1x+a_2=0$. Then the relations will be

$$\alpha+\beta = -\frac{a_1}{a_0}$$

$$\alpha\beta = \frac{a_2}{a_0}$$

(b) Let α,β,γ be the roots of equation $a_0x^3+a_1x+a_2x+a_3=0$. Then the relations will be

$$\alpha+\beta+\gamma = -\frac{a_1}{a_0}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{a_2}{a_0}$$

$$\alpha\beta\gamma = -\frac{a_3}{a_0}$$

(c) Let $\alpha,\beta,\gamma,\delta$ be the roots of equation $a_0x^4+a_1x^3+a_2x^2+a_3x+a_4=0$. Then the relations will be

$$\alpha+\beta+\gamma+\delta = -\frac{a_1}{a_0}$$

$$\alpha\beta+\beta\gamma+\gamma\delta+\delta\alpha = \frac{a_2}{a_0}$$

$$\alpha\beta\gamma+\beta\gamma\delta+\gamma\delta\alpha+\delta\alpha\beta = -\frac{a_3}{a_0}$$

$$\alpha\beta\gamma\delta = \frac{a_4}{a_0}$$

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Ex1. Solve the equation $x^4+2x^3-21x^2-22x+40=0$, given that the roots being in A.P.

Ans. Let the roots are $\alpha-3d,\ \alpha-d,\ \alpha+d,\ \alpha+3d$.

Then

$$(\alpha-3d)+(\alpha-d)+(\alpha+d)+(\alpha+3d)=-2 \quad \text{——— ①}$$

$$(\alpha-3d)\{(\alpha-d)+(\alpha+d)+(\alpha+3d)\}+(\alpha-d)\{(\alpha+d)+(\alpha+3d)\}+(\alpha+d)(\alpha+3d)=-21 \quad \text{——— ②}$$

$$(\alpha-3d)(\alpha^2-d^2)+(\alpha^2-9d^2)(\alpha-d)+(\alpha^2-9d^2)(\alpha+d)+(\alpha^2-d^2)(\alpha+3d)=22 \quad \text{——— ③}$$

$$(\alpha^2-9d^2)(\alpha^2-d^2)=40 \quad \text{——— ④}$$

From ①

$$\alpha-3\cancel{d}+\alpha-\cancel{d}+\alpha+\cancel{d}+\alpha+3\cancel{d}=-2$$

or, $4\alpha=-2$

or, $\alpha=-\frac{1}{2}$

From (4)

$$(\alpha^2-9d^2)(\alpha^2-d^2)=40$$

or, $\left(\frac{1}{4}-9d^2\right)\left(\frac{1}{4}-d^2\right)=40$

or, $(1-36d^2)(1-4d^2)=640$

or, $1-36d^2-4d^2+144d^4=640$

or, $144d^4-40d^2-639=0$

or, $d^2=\dfrac{40\pm\sqrt{1600-4.144.(-639)}}{2.144}$

or, $d^2=\dfrac{40\pm608}{288}$

$d^2=\dfrac{40+608}{288}$ [Taking +ve sign for real value]

or, $d^2=\dfrac{\cancel{648}^{\,9}}{\cancel{288}_{\,4}}=\dfrac{9}{4}$

or, $d=\pm\frac{3}{2}$

Taking $d=\frac{3}{2}$ the roots are

$\alpha-3d,\quad \alpha-d,\quad \alpha+d,\quad \alpha+3d$

$-\frac{1}{2}-\frac{9}{2},\ -\frac{1}{2}-\frac{3}{2},\ -\frac{1}{2}+\frac{3}{2},\ -\frac{1}{2}+\frac{9}{2}$

$-5,\ -2,\ 1,\ 4$

Taking $d=-\frac{3}{2}$ the roots are

$\alpha-3d,\ \alpha-d,\ \alpha+d,\ \alpha+3d$

$-\frac{1}{2}+\frac{9}{2},\ -\frac{1}{2}+\frac{3}{2},\ -\frac{1}{2}-\frac{3}{2},\ -\frac{1}{2}-\frac{9}{2}$

$4,\ 1,\ -2,\ -5$

Hence the required roots are $-5, -2, 1, 4$.

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Ex2. Solve the equation $3x^3-26x^2+52x-24=0$ given that the roots being in G.P.

Ans. Let the roots are $\frac{\alpha}{\rho}, \alpha$ and $\alpha\rho$.

Then
$$\alpha\left(\frac{1}{\rho}+1+\rho\right)=\frac{26}{3} \quad \text{—— (1)}$$

$$\frac{\alpha}{\rho}.\alpha.\alpha\rho=\frac{24}{3} \quad \text{—— (2)}$$

from (2) $\alpha^3=8$
$\alpha=2$

From (1) $2\left(\frac{1}{\rho}+1+\rho\right)=\frac{26}{3}$

or $\frac{3(1+\rho+\rho^2)}{\rho}=13$

or, $3+3\rho+3\rho^2-13\rho=0$

or, $3\rho^2-10\rho+3=0$

or, $3\rho^2-9\rho-\rho+3=0$

or, $3\rho(\rho-3)-(\rho-3)=0$

or, $(\rho-3)(3\rho-1)=0$

or, $\rho=3, \frac{1}{3}$

If $\rho=3$ the roots are
$\frac{\alpha}{\rho}, \alpha, \alpha\rho$
$\frac{2}{3}, 2, 6$

If $\rho=\frac{1}{3}$ the roots are
$\frac{\alpha}{\rho}, \alpha, \alpha\rho$
$6, 2, \frac{2}{3}$

Hence, the required roots are $\frac{2}{3}, 2, 6$.

Ex3. If the roots of the equation $x^3+3px^2+3qx+r=0$ be in H.P., then show that $2q^3=r(3pq-r)$.

Ans. Let α, β, γ be the roots of the given equation.

Then
$$\alpha+\beta+\gamma=-3p \quad \text{—— (1)}$$
$$\alpha\beta+\beta\gamma+\gamma\alpha=3q \quad \text{—— (2)}$$
$$\alpha\beta\gamma=-r \quad \text{—— (3)}$$

Also given $\frac{1}{\alpha}+\frac{1}{\gamma}=\frac{2}{\beta}$ [$\because \alpha, \beta, \gamma$ are in H.P.]

$$\Rightarrow \alpha\beta+\beta\gamma=2\alpha\gamma \quad \text{—— (4)}$$

From ② & ④ we get

$$3\alpha\gamma = 3q$$

or, $\alpha\gamma = q$ ——— ⑤

By ③ ÷ ⑤ we get

$$\frac{\alpha\beta\gamma}{\alpha\gamma} = -\frac{r}{q}$$

or, $\beta = -\frac{r}{q}$ ——— ⑥

From equation ④

$$\beta(\alpha+\gamma) = 2q$$

or, $-\frac{r}{q}(\alpha+\gamma) = 2q$ [using eqⁿ ⑥]

or, $\alpha+\gamma = -\frac{2q^2}{r}$ ——— ⑦

From ①

$$\alpha+\beta+\gamma = -3p$$

or, $\alpha+\gamma = -3p-\beta$

or, $-\frac{2q^2}{r} = -3p-\left(-\frac{r}{q}\right)$ [using eqⁿ ⑦]

or, $\frac{2q^2}{r} = +\left(3p-\frac{r}{q}\right)$

or, $\frac{2q^2}{r} = \frac{3pq-r}{q}$

or, $2q^3 = r(3pq-r)$ (Proved)

<u>Ex4.</u> If the roots of the equation $x^3-ax^2+x-b=0$ be in H.P., then show that the mean root is $3b$.

<u>Ans.</u> Let α, β, γ be the roots of the given equation.

Then $\alpha+\beta+\gamma = a$ ——— ①

$\alpha\beta+\beta\gamma+\gamma\alpha = 1$ ——— ②

$\alpha\beta\gamma = b$ ——— ③

Also $\frac{1}{\alpha}+\frac{1}{\gamma} = \frac{2}{\beta}$ [$\because \alpha, \beta, \gamma$ are in H.P]

or, $\alpha\beta+\beta\gamma = 2\alpha\gamma$ ——— ④

From eq ② and ④

$$3\alpha\gamma = 1$$

or, $\alpha\gamma = \frac{1}{3}$ ——— ⑤

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By ③ ÷ ⑤

$$\frac{\alpha\beta\gamma}{\alpha\gamma} = \frac{b}{\frac{1}{3}}$$

or, $\beta = 3b$ (Proved)

EX5. If the sum of two roots be equal to the third root of the equation $x^3+px^2+qx+r=0$. Then prove that $p^3-4pq+8r=0$

Ans. Let α, β, γ be the roots of the equation

$$x^3+px^2+qx+r=0 \text{ ——— } (1)$$

Then $\alpha+\beta+\gamma = -p$ ——— (2)

$\alpha\beta+\beta\gamma+\gamma\alpha = q$ ——— (3)

$\alpha\beta\gamma = -r$ ——— (4)

Also, according to the given condition

$$\alpha+\beta = \gamma \text{ ——— } (5)$$

From (2) and (5)

$$2\gamma = -p$$

or, $\gamma = -\frac{p}{2}$

Since γ is a root of equation (1) we have

$$\gamma^3+p\gamma^2+q\gamma+r=0$$

or, $\left(-\frac{p}{2}\right)^3+p\left(-\frac{p}{2}\right)^2+q\left(-\frac{p}{2}\right)+r=0$

or, $-\frac{p^3}{8}+\frac{p^3}{4}-\frac{pq}{2}+r=0$

or, $\frac{p^3}{8}-\frac{pq}{2}+r=0$

or, $p^3-4pq+8r=0$ (Proved)

EX6. If one root of the equation $x^3+ax+b=0$ be twice the difference of the other two, then show that one root is $\frac{13b}{3a}$.

Ans. Let α, β, γ is the roots of the given equations

$$x^3+ax+b=0$$

Then $\alpha+\beta+\gamma = 0$ ——— (1)

$\alpha\beta+\beta\gamma+\gamma\alpha = a$ ——— (2)

$\alpha\beta\gamma = -b$ ——— (3)

(5)

Also ~~....~~ according to the given condition

$\alpha = 2(\beta-\gamma)$ —— (4)

From (1) and (4)

$\alpha = 2(\beta-\gamma)$

or, $-\beta-\gamma = 2\beta-2\gamma$

or, $\gamma = 3\beta$ —— (5)

From (2) and (5)

$\alpha(\beta+\gamma)+\beta\gamma = a$

or, $-(\beta+\gamma)^2+\beta\gamma = a$ [using eq^n (1)]

or, $-(\beta+3\beta)^2+\beta.3\beta = a$

or, $-16\beta^2+3\beta^2 = a$

or, $-13\beta^2 = a$

or, $\beta^2 = -\frac{a}{13}$ —— (6)

By (3) ÷ 6

$\frac{\alpha\beta\gamma}{\beta^2} = \frac{-b}{-\frac{a}{13}}$

or, $\frac{\alpha.\beta.3\beta}{\beta^2} = \frac{13b}{a}$

or, $3\alpha = \frac{13b}{a}$

or, $\alpha = \frac{13b}{3a}$ (Proved)

Ex7. If the equation $ax^3+3bx^2+3cx+d=0$ has two equal roots, then show that $(bc-ad)^2 = 4(b^2-ac)(c^2-bd)$ and each of the equal roots is $\frac{bc-ad}{2(ac-b^2)}$.

Ans. Let α, α, β be the roots of the given equation $ax^3+3bx^2+3cx+d=0$ —— (1)

Then $\alpha+\alpha+\beta = -\frac{3b}{a} \Rightarrow 2\alpha+\beta = -\frac{3b}{a}$ —— (2)

$\alpha.\alpha+\alpha\beta+\alpha\beta = \frac{3c}{a} \Rightarrow \alpha^2+2\alpha\beta = \frac{3c}{a}$ —— (3)

$\alpha^2\beta = -\frac{d}{a}$ —— (4)

Applying (2) $\times 2\alpha$ − (3)

$$\begin{aligned} 4\alpha^2+\cancel{2\alpha\beta} &= -\frac{6b}{a}\alpha \\ \underline{-}\,\alpha^2 \underline{\pm}\, \cancel{2\alpha\beta} &= \underline{-}\,\frac{3c}{a} \\ \hline 3\alpha^2 &= -\frac{6b}{a}\alpha-\frac{3c}{a} \end{aligned}$$

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or, $3\alpha^2 a = -6b\alpha - 3c$

or, $a\alpha^2 + 2b\alpha + c = 0$ ———— (5)

From (2) and (4)

$$\alpha^2\beta = -\frac{d}{a}$$

or, $\alpha^2\left(-\frac{3b}{a} - 2\alpha\right) = -\frac{d}{a}$

or, $\alpha^2(3b + 2a\alpha) = d$

or, $2a\alpha^3 + 3b\alpha^2 - d = 0$ ———— (6)

Applying (5) $\times 2\alpha$ − (6)

$$\begin{array}{r} \cancel{2a\alpha^3} + 4b\alpha^2 + 2c\alpha = 0 \\ \cancel{2a\alpha^3} + 3b\alpha^2 - d = 0 \\ - \quad - \quad + \quad \\ \hline b\alpha^2 + 2c\alpha + d = 0 \end{array}$$ ———— (7)

From (5) and (7), we get by cross-multiplication,

$$\frac{\alpha^2}{2bd - 2c^2} = \frac{\alpha}{cb - ad} = \frac{1}{2ac - 2b^2}$$

$$\therefore \quad \alpha = \frac{bc - ad}{2(ac - b^2)}$$

Also $\quad \alpha^2 = \frac{2bd - 2c^2}{2ac - 2b^2}$

or, $\frac{(bc-ad)^2}{4(ac-b^2)^2} = \frac{\cancel{2}(bd - c^2)}{\cancel{2}(ac - b^2)}$

or, $(bc-ad)^2 = 4(bd - c^2)(ac - b^2)$

or, $(bc-ad)^2 = 4(b^2 - ac)(c^2 - bd)$ (Proved)

Ex8. If the sum of two roots of the equation $2x^3 - 3x^2 + kx - 1 = 0$ be one, then find the value of k. Also find the roots of the equation.

Ans. Let α, β, γ be the roots of the equation

$$2x^3 - 3x^2 + kx - 1 = 0 \quad \text{———— (1)}$$

Given $\alpha + \beta = 1$ ———— (2)

$\alpha + \beta + \gamma = \frac{3}{2}$ ———— (3)

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{k}{2}$ ———— (4)

$\alpha\beta\gamma = \frac{1}{2}$ ———— (5)

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from ② & ③

$$1+\gamma = \frac{3}{2}$$

or, $\gamma = \frac{3}{2} - 1$

or, $\gamma = \frac{1}{2}$ ——— ⑥

from ⑤ and ⑥

$$\alpha\beta\left(\frac{1}{2}\right) = \frac{1}{2}$$

or, $\alpha\beta = 1$ ——— ⑦

from ②

$$\alpha+\beta = 1$$

or, $\alpha + \frac{1}{\alpha} = 1$

or, $\alpha^2 + 1 = \alpha$

or, $\alpha^2 - \alpha + 1 = 0$

or, $\alpha = \frac{1 \pm \sqrt{1-4}}{2}$

or, $\alpha = \frac{1 \pm i\sqrt{3}}{2}$

$\therefore$ $\alpha = \frac{1+i\sqrt{3}}{2}$

$\beta = \frac{1}{\alpha} = \frac{1-i\sqrt{3}}{2}$

from ④

$$\alpha\beta + \gamma(\alpha+\beta) = \frac{k}{2}$$

or, $1 + \gamma = \frac{k}{2}$ [using ② & ⑦]

or, $1 + \frac{1}{2} = \frac{k}{2}$ [using 6]

or, $k = \frac{3}{\cancel{2}} \times \cancel{2}$

or, $k = 3$

Hence, the value of k is 3 and the roots of the equation are $\frac{1+i\sqrt{3}}{2}$, $\frac{1-i\sqrt{3}}{2}$, $\frac{1}{2}$.

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Ex 9. If the equation $x^4-px^3+qx^2-rx+s=0$ has two roots of the form $(\alpha+i\alpha)$, $(\beta+i\beta)$, where α and β are real, then show that

$$p^2=2q \text{ and } r^2=2qs$$

Hence, or otherwise solve the equation

$$x^4+4x^3+8x^2-120x+900=0.$$

Ans. Let $\alpha+i\alpha$, $\alpha-i\alpha$, $\beta+i\beta$, $\beta-i\beta$ are the roots of the equation

$$x^4-px^3+qx^2-rx+s=0$$

$$(\alpha+i\alpha)+(\alpha-i\alpha)+(\beta+i\beta)+(\beta-i\beta)=p \Rightarrow 2(\alpha+\beta)=p \quad \text{——— (1)}$$

$$2\alpha^2+(\alpha+i\alpha)(\beta+\cancel{i\beta}+\beta-\cancel{i\beta})+(\alpha-i\alpha)(\beta+\cancel{i\beta}+\beta-\cancel{i\beta})+2\beta^2=q$$

or, $2\alpha^2+2\beta(\alpha+\cancel{i\alpha}+\alpha-\cancel{i\alpha})+2\beta^2=q$

or, $2\alpha^2+4\alpha\beta+2\beta^2=q$

or, $\alpha^2+2\alpha\beta+\beta^2=\frac{q}{2}$

or, $(\alpha+\beta)^2=\frac{q}{2}$ ——— (2)

or $\frac{p^2}{4}=\frac{q}{2}$ or $p^2=2q$ (Proved)

$$2\alpha^2(\beta+i\beta)+2\alpha^2(\beta-i\beta)+2\beta^2(\alpha+i\alpha)+2\beta^2(\alpha-i\alpha)=r$$

or, $2\alpha^2(\beta+\cancel{i\beta}+\beta-\cancel{i\beta})+2\beta^2(\alpha+\cancel{i\alpha}+\alpha-\cancel{i\alpha})=r$

or, $4\alpha^2\beta+4\beta^2\alpha=r$

or, $\alpha\beta(\alpha+\beta)=\frac{r}{4}$ ——— (3)

$$(\alpha+i\alpha)(\alpha-i\alpha)(\beta+i\beta)(\beta-i\beta)=s$$

or, $(\alpha^2+\alpha^2)(\beta^2+\beta^2)=s$

or, $4\alpha^2\beta^2=s$

or, $\alpha^2\beta^2=\frac{s}{4}$ ——— (4)

Now from eq (3)

$\alpha^2\beta^2(\alpha+\beta)^2=\frac{r^2}{16}$

or, $\frac{s}{4}\cdot\frac{q}{2}=\frac{r^2}{16}$ [using eq (2) and (4)]

or $r^2=2qs$ (Proved)

2nd part

We have equation

$$x^4+4x^3+8x^2-120x+900=0 \quad \text{——— (5)}$$

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from eq^n (1)

$$2(\alpha+\beta) = -4$$

or, $\alpha+\beta = -2$ ——— (6)

from eq^n (4)

$$\alpha^2\beta^2 = \frac{900}{4} = 225$$

or, $\alpha\beta = \pm\sqrt{225}$

or $\alpha\beta = \pm 15$

If $\alpha\beta = 15$

$\alpha(-2-\alpha) = 15$

or, $-2\alpha-\alpha^2 = 15$

or, $\alpha^2+2\alpha+15 = 0$

or, $\alpha = \frac{-2\pm\sqrt{4-4.15}}{2.1}$

α is real. So, we can not take this value.

If $\alpha\beta = -15$

$\alpha(-2-\alpha) = -15$

or, $-2\alpha-\alpha^2 = -15$

or $2\alpha+\alpha^2 = 15$

or $\alpha^2+2\alpha-15 = 0$

or $\alpha^2+5\alpha-3\alpha-15 = 0$

or $\alpha(\alpha+5)-3(\alpha+5) = 0$

or, $(\alpha+5)(\alpha-3) = 0$

or, $\alpha = -5, 3$

from (6) if $\alpha = -5$

$\alpha+\beta = -2$

or, $\beta = -2-\alpha$

or, $\beta = -2-(-5)$

or, $\beta = 3$

If $\alpha = 3$ then from eq^n (6)

$\beta = -2-\alpha$

$= -2-3$

$= -5$

If $\alpha = -5, \beta = 3$ then

$\alpha\pm i\alpha$, $\beta\pm i\beta$

$-5\mp i5$, $3\pm 3i$

If $\alpha = 3, \beta = -5$ then

$\alpha+i\alpha$, $\beta\pm i\beta$

$3+3i$, $-5\mp 5i$

Hence, the required roots are $3\pm 3i$, $-5\pm 5i$.

Ex 10. If α be a multiple root of order 3 of the equation

$$x^4+bx^2+cx+d=0,$$

then show that $\alpha = -\frac{8d}{3c}$.

(Ans) Let $\alpha, \alpha, \alpha, \beta$ are the roots of the equation

$$x^4+bx^2+cx+d=0 \quad \text{——— (1)}$$

Then $\alpha+\alpha+\alpha+\beta=0 \Rightarrow 3\alpha+\beta=0$ ——— (2)

$\alpha.\alpha+\alpha.\alpha.+\alpha.\beta+\alpha.\alpha+\alpha.\beta+\alpha.\beta=b \Rightarrow 3\alpha^2+3\alpha\beta=b$ ——— (3)

$\alpha.\alpha.\alpha+\alpha.\alpha.\beta+\alpha.\alpha.\beta+\alpha.\alpha.\beta=-c \Rightarrow \alpha^3+3\alpha^2\beta=-c$ ——— (4)

$\alpha^3\beta=d$ ——— (5)

from eq (2) $\beta=-3\alpha$ ——— (6)

from eq^n (3) $3\alpha^2+3\alpha\beta=b$

$3\alpha^2+3\alpha(-3\alpha)=b$

or, $3\alpha^2-9\alpha^2=b$

or, $-6\alpha^2=b$ ——— (7)

or, $\alpha^2=-\frac{b}{6}$

from eq^n (4) $\alpha^3+3\alpha^2(-3\alpha)=-c$

or, $\alpha^3-9\alpha^3=-c$

or, $-8\alpha^3=-c$

or, $\alpha^3=\frac{c}{8}$ ——— (8)

from eq^n (5) $\alpha^3\beta=d$

or, $\alpha^3(-3\alpha)=d$

or, $-3\alpha^4=d$ ——— (9)

or, $\alpha^4=-\frac{d}{3}$

Applying (9) ÷ (8)

$$\frac{\alpha^4}{\alpha^3}=\frac{-\frac{d}{3}}{\frac{c}{8}}$$

or, $\alpha=-\frac{8d}{3c}$ (Proved)

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Exercise

1. Solve the equation $x^3-9x^2+23x-15=0$, given that the roots are in A.P. [Ans $1, 3, 5$]

2. Solve the equation $27x^3+42x^2-28x-8=0$, given that the roots are in G.P. [Ans $-\frac{2}{9}, \frac{2}{3}, -2$]

3. Solve the equation $2x^3-x^2-22x-24=0$, two of the roots being in the ratio $3:4$. [Ans. $-\frac{3}{2}, -2, 4$]

4. Solve the equation $24x^3+46x^2+9x-9=0$, one of the roots being double of another. [Ans. $-\frac{3}{4}, -\frac{3}{2}, \frac{1}{3}$]

5. Solve the equation $x^3-5x^2-16x+80=0$, if two of its roots α and β be connected by the relation $\alpha+\beta=0$. [Ans. $4, -4, 5$]

6. Determine r so that one of root of the equation
$$x^3-rx^2+rx-4=0$$
shall be reciprocal of another and find the roots. [Ans. $r=5$; $4, \frac{1}{2}(1 \pm i\sqrt{3})$]

7. Solve the equation $x^4+4x^3-2x^2-12x+9=0$ which has two pairs of equal roots. [Ans. $1, 1, -3, -3$]

8. If $\alpha, \beta, \gamma, \delta$ be the roots of the equation
$$x^4-ax^3+bx^2-cx-1=0,$$
find the condition that $\alpha\beta-\gamma\delta=2$. [Ans. $a^2=c^2+4b$]

9. Find the condition that the roots of the equation
$$x^4+px^3+qx^2+rx+s=0$$
~~are~~ are in G.P. [Ans. $p^2s=r^2$]

10. Find the condition that the roots of the equation $x^3+px^2+qx+r=0$ are in A.P. [Ans. $p(9q-2p^2)=27r$]