Graphical or Geometrical Method of Solving a L.P.P.

We know that there is one-one correspondence between the basic feasible solutions (BFS) and extreme points of the convex set of feasible solutions (in the absence of degeneracy).

There are two types of convex set of feasible solutions (1) convex polyhedron and (2) convex polytope.

Convex polyhedron	Convex polytope
(i) has finite number of extreme points and is strictly bounded.	(i) has finite number of extreme points but not bounded from above.
(ii) Every problem has both finite maximum and finite minimum.	(ii) may have finite maximum or finite minimum but not both. Some problem has neither finite maximum nor finite minimum and the problem has said to have unbounded solution.

Ex. Find the convex set of feasible solutions and its nature. Find the feasible space or feasible region.

(a) Maximize $2x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 3$

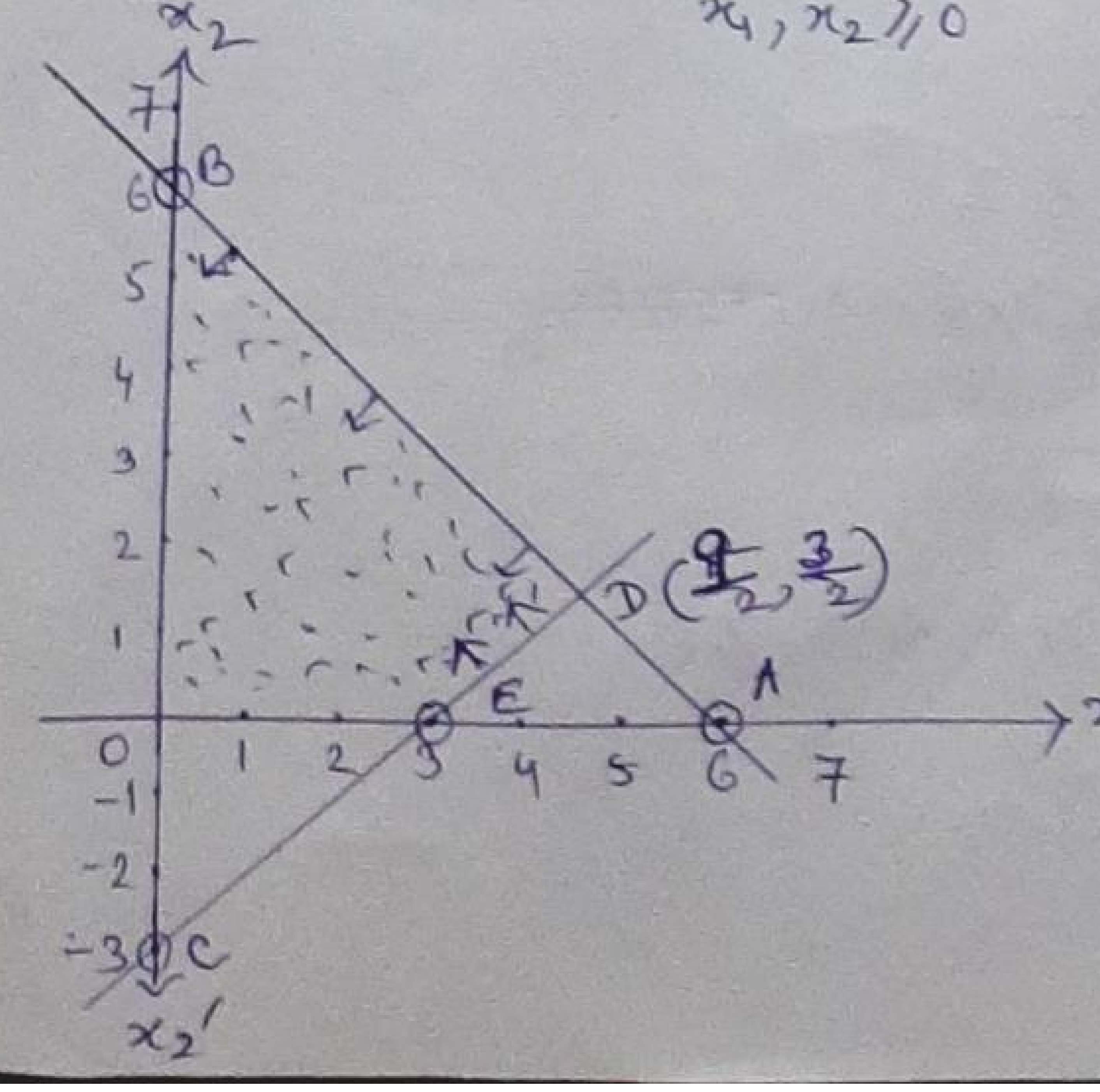
$x_1, x_2 \geq 0$

(b) Maximize $3x_1 + x_2$

Subject to $x_1 - x_2 \leq 2$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$



Suppose Ox_1 and Ox_2 are two mutually $\perp^r$ axes. We have

$$x_1 + x_2 = 6$$

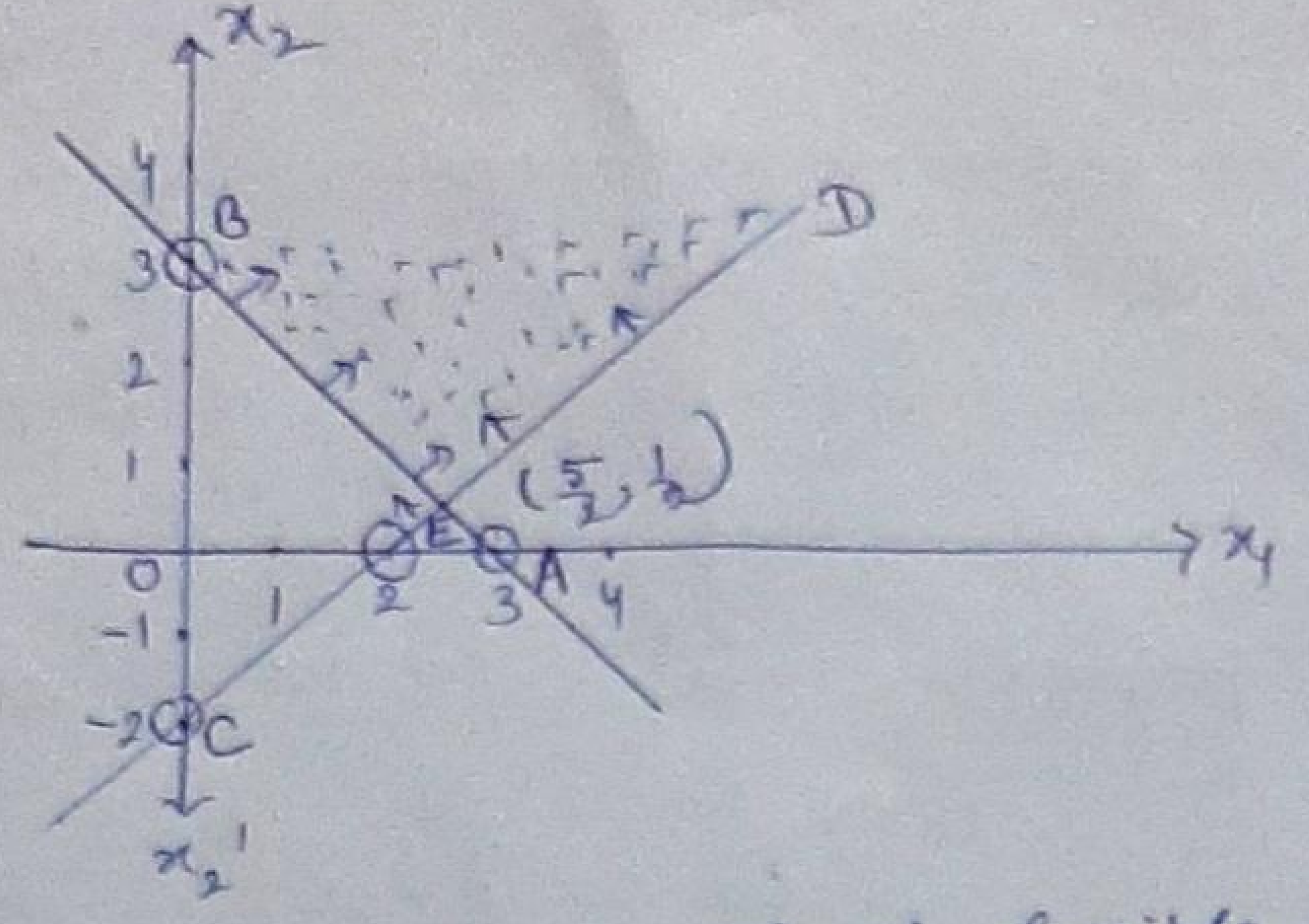
$$\frac{x_1}{6} + \frac{x_2}{6} = 1 \quad \text{—(1)}$$

and $x_1 - x_2 = 3$

$$\frac{x_1}{3} + \frac{x_2}{(-3)} = 1 \quad \text{—(2)}$$

$\overline{AB}$ and $\overline{CD}$ represents the st. lines ~~found~~ written in eq^n (1) and 2 respectively.

Since $x_1, x_2 \geq 0$, then convex set of the feasible region is OEDBO and the extreme points are $O(0,0)$, $E(3,0)$, $D(\frac{9}{2},\frac{3}{2})$, $B(0,6)$. The convex set is a convex polyhedron. The shaded region OEDBO is the feasible space or feasible region.



$x_1 - x_2 \leq 2$

$\frac{x_1}{2} + \frac{x_2}{(-2)} = 1$ — (1)

and $x_1 + x_2 = 3$

$\frac{x_1}{3} + \frac{x_2}{3} = 1$ — (2)

Here the convex set of feasible solution (F.S.) has two extreme points $E(\frac{5}{2},\frac{1}{2})$ and $B(0,3)$. Here the convex set of F.S. is not a convex polyhedron as it is not possible to express all points of the feasible set as the convex combination of extreme points $E(\frac{5}{2},\frac{1}{2})$ and $B(0,3)$. It is a convex polytope.

The shaded region is the feasible space or feasible region.

Ex. Find feasible region of the following L.P.P. Also find out the extreme points of the feasible region.

(i) Maximize $Z = 5x_1 + 3x_2$

Subject to $x_1 + 2x_2 \leq 7$

$x_1 - x_2 \leq 5$, $x_1, x_2 \geq 0$

(ii) Minimize $z = 2x_1 - 3x_2$

Subject to $x_1 + 2x_2 \leq 2$

$3x_1 + 2x_2 \geq 6$, $x_1, x_2 \geq 0$

(iii) Maximize $z = x_1 + 4x_2$

Subject to $x_1 + x_2 \geq 5$

$x_1 + 3x_2 \geq 6$, $x_1, x_2 \geq 0$

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(4) Minimize $Z = -3x_1 + x_2$

Subject to $x_1 + x_2 \geq 4$

$5x_1 + 2x_2 \leq 10$, $x_1, x_2 \geq 0$

Ex. Solve the following L.P.P. graphically.

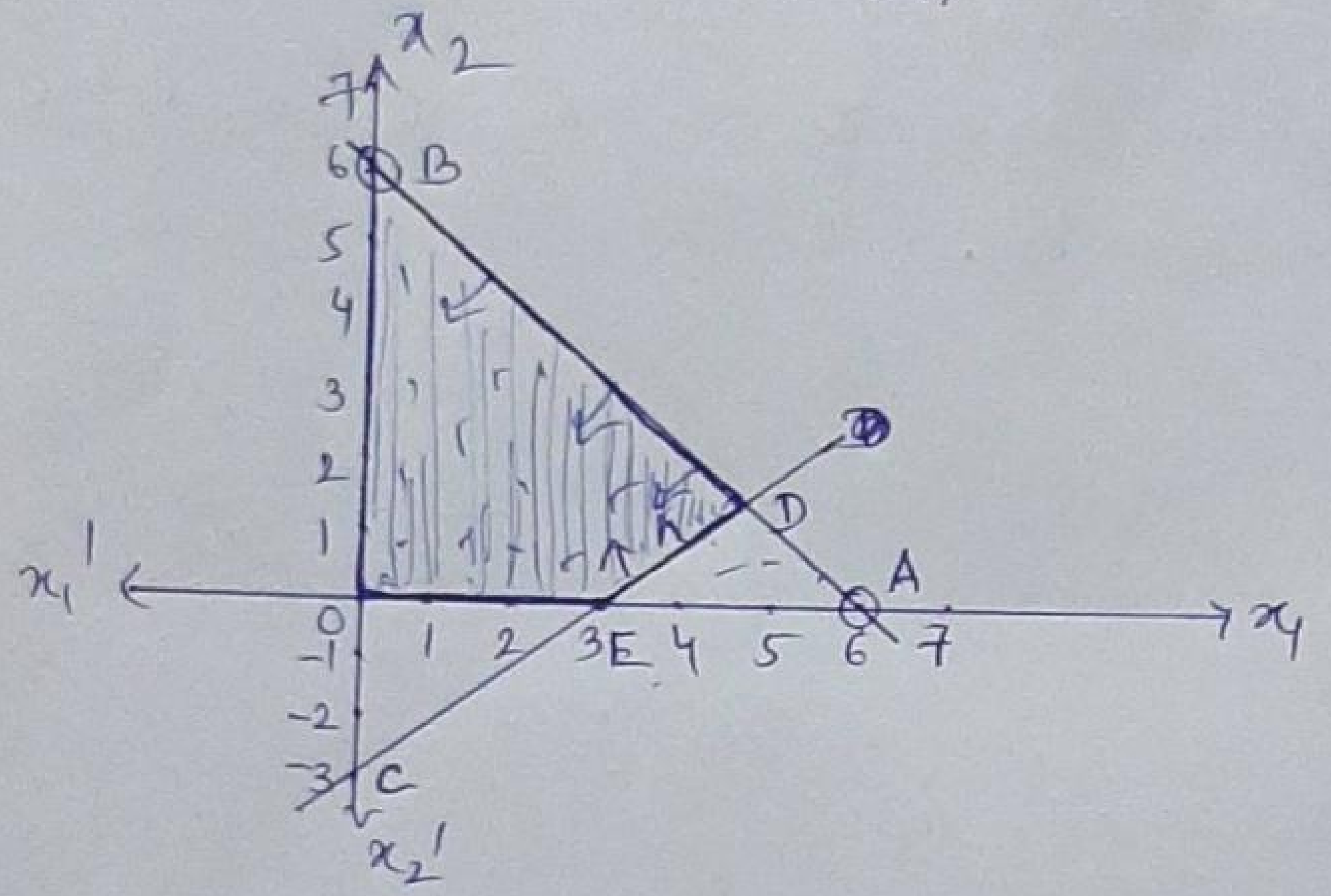
Maximize $Z = 2x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 3$

$x_1, x_2 \geq 0$

Ans



Suppose Ox_1 and Ox_2 are two mutually perpendicular axes. The given inequations can be written as

$$x_1 + x_2 = 6$$

$$\frac{x_1}{6} + \frac{x_2}{6} = 1 \quad \text{--- (1)}$$

and

$$x_1 - x_2 = 3$$

$$\frac{x_1}{3} + \frac{x_2}{(-3)} = 1 \quad \text{--- (2)}$$

AB and CD represents the straight lines written in equations (1) and (2) respectively.

From ~~Solving eq (1) &~~ (2) we get

$$x_1 = 3 + x_2$$

from (1) $3 + x_2 + x_2 = 6$

$$2x_2 = 6 - 3$$

or $x_2 = \frac{3}{2}$

$$\therefore x_1 = 3 + \frac{3}{2} = \frac{9}{2}$$

∴ the feasible region is OEDBO with the extreme points $O(0,0)$, $E(3,0)$, $D(\frac{9}{2},\frac{3}{2})$, $B(0,6)$

Now

$$[Z]_O = 2.0 + 3.0 = 0$$

$$[Z]_E = 2.3 + 3.0 = 6$$

$$[Z]_D = 2.\frac{9}{\cancel{2}} + 3.\frac{3}{2} = 9 + \frac{9}{2} = 13\frac{1}{2}$$

$$[Z]_B = 2.0 + 3.6 = 18$$

∴ The optimal solutions are $x_1 = 0$, $x_2 = 6$ and the corresponding Max $Z = 18$.