Assigments on Actuarial Statistics

1 Utility Theory

1. What is an utility function? An insurer, whose current wealth is W, uses the utility function

$$u(x) = x - \frac{x^2}{2\beta} \qquad x < \beta$$

for decision making purpose. Show that the insurer is the risk averse and the insurer's risk aversion coeffcient, $r(x)$, is an increasing function of x.

2. An insurer has been asked to provide complete insurance cover against a random loss X where $X \sim N(10^6, 10^8)$. Calculate the minimum premium that the insurer would accept if the insurer bases on the utility function $u(x) = -\exp\{-0.002x\}$.

3. An individual is facing a random loss, X, that is uniformly distributed on $(0, 200)$. The individual can buy partial insurance cover against this loss under which the individual would pay $Y = \min(X, 100)$, so that individual would pay loss in full if the loss is less than 100 and would pay 100 otherwise. The individual makes decisions using utility function $u(x) = x^{2/5}$. Is the individual prepared to pay premium 80 for this partial insurance cover if the individual's wealth is 300?
Hint: The individual is prepared to pay 80 *for this partial insurance cover if*

$$E(u(300 - X)) \leq E(u(300 - 80 - Y))$$

4. An individual and an insurance company model the utility of their wealth by the function $U(x) = -\exp(-0.002x)$ and $U(x) = -\exp(-0.001x)$ respectively. Both may face a random loss X, which has an exponential distribution with mean 50 units. Find the maximum individual will may for complete insurance and minimum acceptable premium to the insurance company. Is the insurance contract is feasible?

5. An insurer is considering offering complete insurance cover against a random loss X, where $E(X) = \text{Var}(X) = 100$ and $P(X > 0) = 1$The insurer adopts utility function $u(x) = x - 0.001x^2$ for decision purpose. Calculate the minimum premium that the insurer would accept for this insurance cover when the insurer's wealth is 300.

6. Each of the following four decision makers faces a random loss with uniform distribution over the interval $(0, 10)$ and each has same utility function given by $U(x) = \sqrt{x}$ for $x \geq 0$. The wealth of the decision makers are given below

Decision maker	A	B	C	D
Wealth	10	15	20	25

   Who will purchase complete insurance at a premium of 5.2?

7. A decision maker uses utility function $U(x) = \ln x, x > 0$ and wealth Rs 10 lakhs. He is willing to pay a premium of Rs $50,000/-$ to cover a risk with probabilities $3/4$ of no loss and $1/4$ of loss L. Determine the value of L.

2 Principles of premium calculation

8. What are the desirable properties of premium principles? Explain each.

9. Show that expected value principle, the variance principle and the standard deviation principle do not satisfy the *no-ripoff* property.

10. Show that principle of zero utility is consistent and it satisfies the non negative loading and no-ripoff properties.

11. Let X_1 has probability mass function

$$P(X_1 = 80) = 0.5 = 1 - P(X_1 = 120)$$

and let X_2 has probability mass function

$$P(X_2 = 90) = 0.6 = 1 - P(X_2 = 120)$$

An insurer has wealth 300 and calculates premiums using principle of zero utility with utility function

$$u(x) = x - 0.001x^2$$

for $x < 500$. Calculate premiums under the distribution of X_1, X_2 and $X_1 + X_2$ and hence verify principle of zero utility is not additive in general.

12. Let $X \sim Gamma(2, 0.01)$. The pdf $X \sim Gamma(\alpha, \lambda)$ is given by

$$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} e^{-\lambda x} x^{\alpha-1} \qquad x > 0.$$

Given that premium is $\prod_X = 250$ and $\prod_X$ has been calculated using by Esscher principle with parameter h, calculate h.

13. Let $X \sim U(5, 15)$. Calculate premium using risk adjusted premium principle with risk index 1.2.

14. The mean value principle states that the premium, Π_X for the risk X is given by

$$\Pi_X = v^{-1} E(v(X))$$

where v is a function such that $v'(x) > 0$ and $v''(x) \geq 0$ for $x > 0$. Calculate Π_X when $v(x) = x^2$ and $X \sim Gamma(2, 2)$.

3 Individual Risk Model

15. Define Individual risk model. How can you approximate the distribution of total claim under individual risk model.

16. The portfolio of an insuarance company consists of 350 policies. For each policy the probability q of a claim is 0.002 and the benefit amount, given that there is a claim, has gamma distribution with scale parameter 0.25 and shape parameter 50. Suppose S denotes the total claims for portfolio for a short period. Find $P(S > 71000)$, $P(S < 69,000)$ and $P(69,000 < S < 71,000)$. What relative security loading , θ, should be used so the company can collect an amount of 99th percentile of the distribution of total claims.

17. The following table shows the data for a life insurance company which which issues one year term life contracts

Group	Death Rate	Benefit Amount	Number of lives
1	0.001	1	100
2	0.002	2	300
3	0.003	3	400
4	0.003	4	350

Find the expectation and variance of the total risk from the portfolio to the company. The company wants to collect, from these individuals, an amount equal to the 95^{th} percentile of the distribution of total claims. The share for individual j with mean $E(X_j)$ would be $(1+\theta)E(X_j)$. Calculate the relative security loading θ.

18. The policyholders in an automobile company fall into two classes

Class	Number	Claim	Distribition of claim amount	
	in class	Probability	Parameters of Truncated exponential	
k	n_k	q_k	λ	L
1	500	0.10	1	2.5
2	2000	0.05	2	5

The truncated exponential distribution is defined by the df

$$F(x) = \begin{cases} 0 \quad \text{if} \quad x < 0 \\ 1 - e^{-\lambda x} \quad \text{if} \quad 0 \leq x < L \\ 1 \quad \text{if} \quad x \geq L \end{cases}$$

This is mixed distribution with p.d.f. $f(x) = \lambda e^{-\lambda x}$ if $0 < x < L$ and mass $e^{-\lambda L}$ at L. Again the probability that total claim exceed the amount collected from policy holders is 0.05. Moreover, it wants each individual's share of this amount to be proportional to individual's expected claim. The share of the individual j with mean $E(X_j)$ would be $(1+\theta)E(X_j)$. We assume that relative security loading θ is same for two classes. Calculate θ.

19. Consider a portfolio of 32 policies. For each policy, the probability q of a claim is 1/6 and B, the benefit amount given that there is a claim, has p.d.f.

$$f(y) = \begin{cases} 2(1-y) \qquad 0 < y < 1 \\ 0 \qquad \text{otherwise} \end{cases}$$

Let S be the total claims for the portfolio.Using normal approximation estimate $P(S > 4)$.

4 Collective Risk Model

20. When collective risk model is used? Obtain the distribution of total claim S under collective risk model.

21. Suppose that S has a compound Poisson distribution with $\lambda = 3$ and the common distribution of $X_1, X_2, \ldots$ is charaterrized by the p.m.f.

$p(x) = 0.1x$ where $x = 1, 2, 3, 4$.Calculate probabilities that aggregate claims equal to $0, 1, 2, 3$. Also, find the probability that aggregate claims exceed 3 units.

22. In a collective risk model, suppose the frequency N of claims is a random variable with possible values $0, 1, 2$ with probabilities $0.4, 0.2, 0.4$ respectively. Amunt X of claims is a random variable with possible values $1000, 3000, 7000$ with probabilities $0.3, 0.4, 0.3$ respectively. Find the distribution of total risk to the company for a short period.

23. For a collective risk model, the number of claims has a negative binomial distribution with parameters $r = 2$ and $p = 1/3$. Claim amounts are mutually independent with two possible values 1 and 2 with equal chance. Find moment generating function of total claims.

24. Aggregate claim S in a collective risk model has a compound negative binomial distribution, with parameters $r = 100$ and $p = 0.3$ for the negative binomial distribution and individual claim distribution is exponential with mean 100 units. Find mean and variance of S. Using normal approximation, find $P[S > 1.3E(S)]$.

25. The frequency of the car accidents follows a Poisson distribution. There are 90% good drivers which on the average commit 1 accident over a specific short period while the similar rate for 10% bad drivers is 3. If an accident occurs, the claim amount has lognormal distribution with location parameter 3 and scale parameter 2. Calculate the mean and variance of the total claims over that short period.