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Hawking decay and thermodynamic transformation of a black hole: two examples

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Abstract

The criterion for thermodynamic stability of rotating electrically charged quantum black holes was already derived by us. They appeared as a collection of inequalities connecting second-order derivatives of the black hole mass with respect to its horizon area, electric charge and angular momentum. We got similar results when this analysis was extended to black holes in arbitrary dimensional spacetime with any number of parameters that determine the mass of the black hole. Many black holes were shown to satisfy some of the stability criteria in certain regions of parameter space, but not all together. They are known as "Quasi Stable" black holes. Quasi stability restricts the accessibility of parameter space; hence, it creates bounds on various parameters of the quasi-stable black holes. They, although decaying under Hawking radiation, possess bounded fluctuations in certain regions of their accessibility for some of their parameters. We here consider Kerr–Newman and Kerr–Sen black holes as examples of two quasi-stable black holes. Their fluctuations are shown to be related to the bounds in parameter space. We also study the decay rate in various regions of their parameter spaces. We conclude that they transform to different kinds of black holes during their Hawking decay.

Keywords Black hole thermodynamics · Quasi-stability · Quantum black holes · Black hole transformation · Phase transition

1 Introduction

The application of Einstein's general theory of relativity concludes that a black hole is capable of accreting anything in its vicinity [1]. Thus, black holes grow in size forever. However, Hawking showed, using his semi-classical theory [2], that black holes were capable of radiating matter. Hence, there is a competition between accretion and radiation within a black hole. Thus, when accretion wins the black hole grows in size forever; otherwise when Hawking radiation wins, the black hole decays. Hence, a black hole reaches its stable equilibrium only when a perfect balance between accretion and radiation exists.

We have already shown, treating spacetime as a quantum mechanical entity, that the stability criteria of a black hole appear as a collection of inequalities, involving the parameters of that black hole [3, 4]. If all the stability criteria hold simultaneously for a black hole within some region of its parameter space, then that black hole is thermodynamically stable in that region. Similarly, a black hole may satisfy the series of stability criteria partially within a certain region of parameter space. Then that black hole is quasi-stable in that region of parameter space. Actually both 'stability' and 'quasi-stability' concern a region of parameter space.

Fluctuations of the parameters of any stable black hole, for example, horizon area, angular momentum and electric charge for charged rotating AdS black holes, have already been calculated [5] by us. They are bounded in the region of stability of the parameter space. In fact, bounded fluctuations, like that of a stable black hole, are observed for some parameters of quasi-stable black holes as well [6, 7]. This is an artifact of the fact that some of their stability criteria are satisfied in certain region of parameter space. Now, quasi-stability restricts the accessible region in the parameter space for such black holes, as beyond this they would be unstable. However, this restriction has certain relations with the fluctuations of their various parameters. We study this here in detail for two quasi-stable black holes, namely a Kerr–Newman black hole (KNBH)and a Kerr–Sen black

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hole (KSBH). We find that both charge and angular momentum have to be reduced for Kerr–Newman black holes, but such applies only to the angular momentum for Kerr–Sen black holes.

That quasi-stable black holes have to decay under Hawking radiation is undoubtedly true , but their similarities with stable black holes in terms of their fluctuations cast a doubt on us, whether quasi-stability can reduce their decay rate. We show that this is indeed true for both KNBH and KSBH. We also show that these fluctuations ultimately help to transform a KNBH into a neutral non-rotating black hole whereas a KSBH is transformed into a non-rotating charged black hole.

This paper is organized as follows. A brief necessary recapitulation of previous works is presented in Sect. 2. The following section shows the details of fluctuations for KSBHs and KNBHs. Thermodynamic transformations of these black holes are also studied here. We draw a connection between the decay rate and quasi-stability of these black holes in Sect. 4. The last section contains a summary with an outlook.

2 Thermal stability and thermal fluctuation

Semiclassical analyses predict thermal instability under Hawking radiation for asymptotically flat black holes due to the negativity of their specific heat [8]. These analyses consider positivity of the specific heat as the sole criterion for thermal stability of any black hole. These semiclassical theories treat only matter as a quantum entity, but black holes are still classical. We had focussed on this issue in our earlier works [3, 4], from a scheme independent perspective, but motivated by some results of LQG [9, 10]. Loop Quantum Gravity (LQG) played only a motivational role there. The work that we had done there, is actually independently of LQG. That work is justified on the ground that LQG might provide situations where these assumptions are valid.

Thus, a study of the thermodynamic stability under Hawking radiation of a rotating charged black hole from this quantum perspective would be interesting. A rotating charged black hole is classically characterized by its mass (M), angular momentum (J) and charge (Q). Thus, we intuitively expect that these parameters will play equally important role in determining the thermodynamic behavior of such black holes, even when analyzed from a quantum perspective. The mass of a charged rotating black hole, on its horizon, depends classically on its horizon area (A), charge (Q) and angular momentum (J). We assume an equivalent relationship for our quantum analysis, i.e.,the mass is a function of the horizon area, angular momentum and charge.

Black holes, at equilibrium, are represented here by isolated horizons that act as internal boundaries of spacetime. The description of the horizon, as given in detail in Ref. [3], helps us to define mass, electric charge, angular momentum, etc. on the horizon. The tensor product of the boundary and the bulk spacetime becomes the Hilbert space for a generic quantum spacetime. By virtue of some physical invariances of bulk spacetime, bulk states are completely decoupled, so only the boundary states determine the whole grand canonical partition function (Z_G) of a black hole if we assume the black hole is in contact with a thermodynamic heat bath that exchanges mass, angular momentum(J) and charge(Q) with the black hole. This has been discussed in detail in Ref. [3]. Thus, Z_G is expressible as [3]

$$Z_G = \sum_{k,l,m} g(k,l,m) \quad \exp\left(-\beta\left(E(A_k,Q_l,J_m) - \Phi Q_l - \Omega J_m\right)\right), \tag{1}$$

where $g(k, l, m)$ is the degeneracy factor associated with the energy eigenvalue $E(A_k, Q_l, J_m)$ and k, m, l are, respectively, the discrete quantum numbers attached to the eigenvalues of area, angular momentum and charge [11]. β, Φ and Ω, respectively, denote the inverse temperature, electric potential and angular speed of the black hole. The spectrum of the boundary Hamiltonian operator is expectantly assumed here to be a function of area, angular momentum and charge of the horizon. Motivated by LQG results, we consider these 'quantum parameters' to have discrete spectra [11, 15]. We, applying the Poisson resummation formula [16], can re-express Z_G in the macroscopic spectra limit of the black hole($k, l, m >> 1$) as,

$$Z_G = \int dA \quad dQ \quad dJ \quad \exp\left[S(A) - \beta(E(A,Q,J) - \Phi Q - \Omega J)\right], \tag{2}$$

where, following Ref. [17], $S(A)$ denotes the microcanonical entropy of the horizon.

The saddle point $(\bar{A}, \bar{Q}, \bar{J})$ represents the black hole in its thermodynamic equilibrium [3]. $\bar{J}$ denotes the angular momentum of the horizon at equilibrium and so on. Z_G is calculated for fluctuations $a = (A - \bar{A})$, $j = (J - \bar{J})$ and $q = (Q - \bar{Q})$ around the saddle point and is given as ([3])

$$Z_G \approx \int da \quad dq \quad dj \quad \exp\left(-\frac{\beta}{2}\left[\left(M_{AA} - \frac{S_{AA}}{\beta}\right)a^2 + (M_{QQ})q^2 + (2M_{AQ})aq + (M_{JJ})j^2 + (2M_{AJ})aj + (2M_{QJ})qj\right]\right), \tag{3}$$

where β denotes the inverse temperature of the black hole.

The stability criteria, equivalent to convexity of the above integral, have been shown to be given as follows [3]:

$$\big(\beta M_{AA} - S_{AA}\big) > 0, M_{QQ} > 0, M_{JJ} > 0$$
$$\big(M_{QQ}M_{JJ} - (M_{JQ})^2\big) > 0, \big(M_{JJ}\big(\beta M_{AA} - S_{AA}\big) - \beta(M_{AJ})^2\big) > 0, \big(M_{QQ}\big(\beta M_{AA} - S_{AA}\big) - \beta(M_{AQ})^2\big) > 0$$
$$|H| > 0,$$

where $|H|$ is the determinant of the Hessian matrix (H), which is given as

$$H = \begin{pmatrix} \beta M_{AA} - S_{AA} & \beta M_{AQ} & \beta M_{AJ} \\ \beta M_{AQ} & \beta M_{QQ} & \beta M_{JQ} \\ \beta M_{AJ} & \beta M_{JQ} & \beta M_{JJ} \end{pmatrix}.$$

Here, $M_{AA} \equiv \frac{\partial^2 M}{\partial A^2}$, $M_{AQ} \equiv \frac{\partial^2 M}{\partial A \partial Q}$ etc., and they are all calculated at the saddle point. The above inequalities are really some inequalities among the parameters of the black hole. The note is that the saddle point is actually the equilibrium point of the black hole. Thus, all the terms M_{AA}, M_{QQ}, etc., are measured at equilibrium. Then, Z_G is evaluated, considering up to Gaussian fluctuations, as in ordinary thermodynamics. Thus, that Z_G is grand canonical partition function within small neighborhood of the equilibrium point is true. If the fluctuations are bounded enough, then the black hole is in stable equilibrium. On the other hand, if at least one fluctuation is unbounded, then the black hole will drift away from the equilibrium point. This, in turn, causes Z_G to diverge and makes the black hole unstable around that equilibrium, i.e., unstable equilibrium. Of course, Z_G cannot predict anything more for unstable black holes, away from the equilibrium point. In fact, we are not interested in anything beyond what Z_G predicts. We should admit the fact that thermodynamics does not give much if we are away from the equilibrium point. We will see later that this Z_G is capable of predicting phase transitions for unstable black holes as well.

The inverse temperature $\beta(= S_A/M_A)$ is taken to be positive. The note is that the entropy of the black hole (S) is taken to be a function of its horizon area (A) only. We see here that there are many stability criterion rather than a sole condition of positivity of specific heat, which is predicted by semiclassical analyses. An asymptotically flat Reissner–Nordstrom black hole (AFRNBH) and some other black holes decay under Hawking radiation in some region of its parameter space in spite of its positive specific heat [18, 20]. Because an AFRNBH is still electrically unstable in that region, it decays.

Some rotating charged black holes satisfy some of the stability criteria, but not all, within a certain region of parameter space. These black holes are quasi-stable within that region. We have already [6, 7] studied asymptotically flat Kerr–Newman (KN) and Kerr–Sen (KS) black holes as examples of quasi-stable black holes. $|H|$ is always negative for them. Hence, they cannot be stable. However, M_{QQ}, M_{JJ}, $\big(M_{QQ}M_{JJ} - (M_{JQ})^2\big)$ are always positive for them. Thus, they are always quasi-stable, irrespective of their position in parameter space.

We already know [6] the mechanism for calculating the fluctuations. $\Delta(J)^2$ measures the fluctuation of angular momentum from its equilibrium value and can be mathematically expressed as [5, 6] $\Delta(J)^2 = \frac{\int \mathrm{d}a\,\mathrm{d}q\,\mathrm{d}j\, j^2 f(a,q,j)}{\int \mathrm{d}a\,\mathrm{d}q\,\mathrm{d}j\, f(a,q,j)}$, where $f(a,q,j) = \exp\Big(-\frac{\beta}{2}[(M_{AA} - \frac{S_{AA}}{\beta})a^2 + (M_{QQ})q^2 + (2M_{AQ})aq + (M_{JJ})j^2 + (2M_{AJ})aj + (2M_{QJ})qj]\Big)$. $\Delta(Q)^2$ and $\Delta(A)^2$ are defined similarly. To calculate and conclude that $\Delta(A)^2$ converges to the value $\frac{\beta^2(M_{QQ}M_{JJ}-(M_{JQ})^2)}{2|H|}$, only if $\frac{(M_{QQ}M_{JJ}-(M_{JQ})^2)}{|H|}$ is positive is easy. Otherwise, $\Delta(A)^2$ would diverge. $\Delta(Q)^2$ and $\Delta(J)^2$ are calculable similarly in a cyclic manner. Thus, we, if $|H|$ is always negative, can conclude the following.

(1) $\Delta(A)^2$ always diverges as $\big(M_{QQ}M_{JJ} - (M_{JQ})^2\big)$ is always positive.
(2) $\Delta(Q)^2$ is bounded only if $\big((\beta M_{AA} - S_{AA})M_{JJ} - \beta(M_{JA})^2\big)$ is negative; otherwise, it diverges.
(3) $\Delta(J)^2$ is bounded only if $\big(M_{QQ}(\beta M_{AA} - S_{AA}) - \beta(M_{AQ})^2\big)$ is negative; otherwise, it diverges.

3 Fluctuations of parameters and thermodynamic transformation of quasi-stable black holes: two examples

We will consider here examples of two quasi-stable, charged, rotating black holes explicitly. We will show in detail how these fluctuations vary with the regions of parameter space. We will conclude thereafter that these fluctuations and some bounds in parameter space together are responsible for possible transformations of these black holes from one type to another type.

3.1 Kerr–Newman black hole

The mass (M) of a KNBH is related to its parameters as [21],
$$M^2 = \frac{A}{16\pi} + \frac{\pi}{A}(4J^2 + Q^4) + \frac{Q^2}{2}$$
Thus, the accessible parameter space is defined by the inequality $\big(4J^2 + Q^4\big) < \frac{A^2}{16\pi^2}$ as the temperature($\propto M_A$) of a non-extremal black hole is always positive. Thus, both the electric charge and the angular momentum are bounded for a given horizon area of the black hole. $|H|$ is always negative; hence, this black hole will decay under Hawking radiation and will consequently lose area. Hence, charge and angular momentum will adjust, through their fluctuations, to maintain the above bound. This bounded region is shown in Fig. 1.

We define the function f_{AQ} as

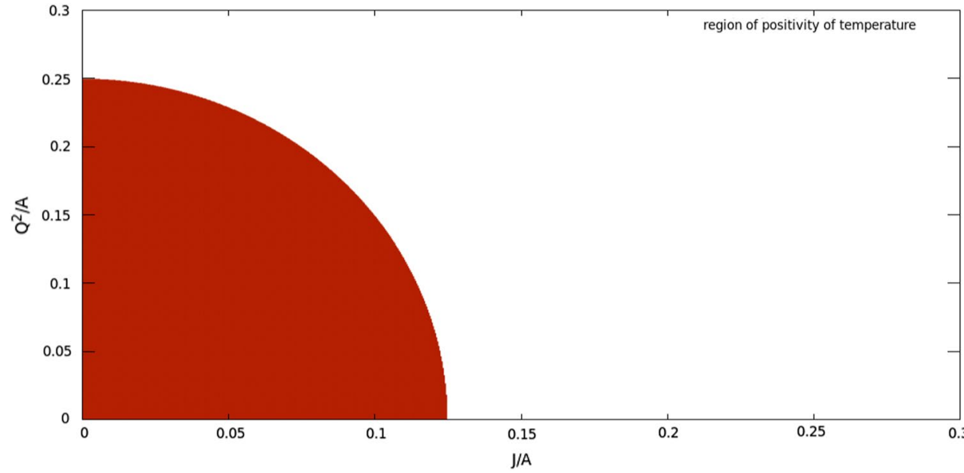

Fig. 1 Pictorial representation of a region of temperature positivity

$$f_{AQ} \equiv \frac{9}{32}y^3 - 34x^2y^3 + 9x^2y^2 + 9x^2y - \frac{7}{8}y^4 - 13y^5 - \frac{3}{64}y^2 - \frac{3}{12}y + \frac{x^2}{16} + 6x^4 + 72x^4y - \frac{1}{2048},$$

where $x \equiv \frac{\pi J}{A}$, $y \equiv \frac{\pi Q^2}{A}$.

Now, that $\left(M_{QQ}(\beta M_{AA} - S_{AA}) - \beta(M_{AQ})^2\right)$ is proportional to f_{AQ} can easily be shown. In Fig. 2, we show the region of parameter space where f_{AQ} is negative, i.e. the region where the fluctuation of angular momentum is bounded. Figure 2 clearly indicates that higher values of $\frac{J}{A}$, of course maintaining the bound, cause the fluctuation of the angular momentum to be large. The area of this black hole always decreases and, consequently, tries to increase the ratio $\frac{J}{A}$. This causes the fluctuation of the angular momentum to be appreciably large; hence, the angular momentum is reduced to maintain the non-extremality bound. Thus, the $\frac{J}{A}$ ratio comes in the region shown in Fig. 2; and hence, J does not fluctuate much. However the area(A), as usual, decreases gradually; consequently, the $\frac{J}{A}$ ratio becomes large enough so that J starts to fluctuate appreciably once again. Thus, this switching of the $\frac{J}{A}$ ratio from a larger to a smaller value and vice versa continues. In this process, angular momentum gradually decreases during Hawking decay. Thus, this KN black hole transforms into a non-rotating black hole.

We now define the function f_{AJ} as
$f_{AJ} \equiv \frac{5}{8}y^2 + 7y^4 + 4y^3 - 16x^2 - \frac{1}{256}$,
where x, y are defined as earlier. Now, that $\left(M_{JJ}(\beta M_{AA} - S_{AA}) - \beta(M_{AJ})^2\right)$ is proportional to f_{AJ} can be shown easily. In Fig. 3, we show the region of parameter space where f_{AJ} is negative, i.e., region where fluctuations of the charge of a black hole are bounded.

Figure 3 shows vividly that higher values of $\frac{Q^2}{A}$ can make the charge fluctuation bounded only if the ratio $\frac{J}{A}$ is high enough. However, we have already shown that the ratio $\frac{J}{A}$ cannot always be high. On top of this, the ratio $\frac{Q^2}{A}$ is itself bounded. Hence, Q reduces gradually during Hawking decay as the black hole always decreases in area. Thus, the ratio $\frac{Q^2}{A}$ oscillates between higher and lower values, as the ratio $\frac{J}{A}$ does, and gradually discharges all its charges. Hence, a

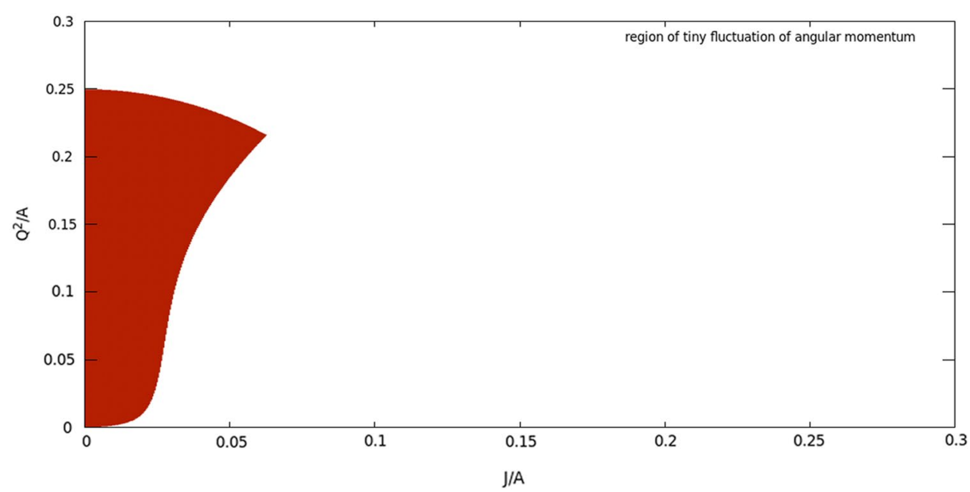

Fig. 2 Pictorial representation of a region of bounded fluctuations of angular momentum

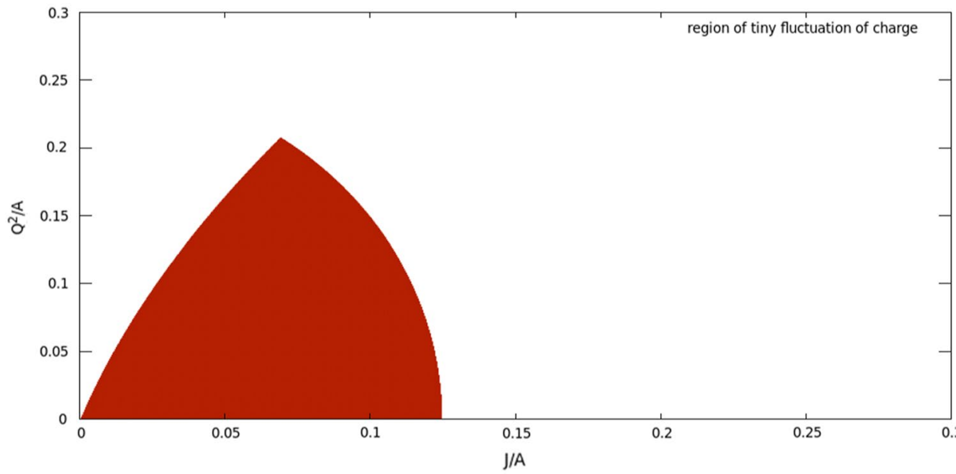

Fig. 3 Pictorial representation of a region of bounded fluctuations of charge

KNBH loses its charge during Hawking decay. As a result, it transforms into a chargeless black hole. Thus, a KN black hole ultimately transforms into a chargeless, non-rotating black hole. Moreover, interestingly, one simple bound on parameter space, namely positivity of the temperature, turns out to determine the end state of a black hole. This makes the thermodynamic analysis robust. Of course, that various non-perturbative quantum gravity-related issues will be important during the black hole's end state is true. A thermodynamic analysis alone, without a study of full-fledged quantum gravity at that stage, cannot predict anything. In fact, this is yet to be done in literature. Nevertheless, we can still predict things close to the end state of black hole correctly just by doing a thermodynamic analysis.

Now, we find that the common region, where both f_{AJ} and f_{AQ} are negative, is the region where both $\Delta(Q)^2$ and $\Delta(J)^2$ are bounded. Figure 4 shows that both $\Delta(J)^2$ and $\Delta(Q)^2$ are bounded when both the ratios $\frac{J}{A}$ and $\frac{Q^2}{A}$ are sufficiently small. In fact, this is the region where both charge and angular momentum stay at their equilibrium values. This region is near the endpoint of the black hole, where it almost loses all its charge and angular momentum. However, at the time, various other quantum gravity factors, as mentioned earlier, may play important roles as well.

3.2 Kerr–Sen black hole

The mass(M) of a KSBH is related to its parameters as [22] $M^2 = \frac{A}{16\pi} + \frac{Q^2}{2} + \frac{4\pi J^2}{A}$.

Thus, the accessible physical parameter space is defined by the inequality $\frac{J}{A} < \frac{1}{8\pi}$ as any non-extremal black hole always has a positive temperature($\propto M_A$). Interestingly the electric charge of this black hole, unlike an AFKNBH, is not bounded by the non-extremality of this black hole. We will see its interesting consequences soon. Now the quantity $\left(M_{QQ}(\beta M_{AA} - S_{AA}) - \beta(M_{AQ})^2\right)$ is negative for $\frac{J}{A} < \frac{0.4}{8\pi}$ whereas $\left((\beta M_{AA} - S_{AA})M_{JJ} - \beta(M_{JA})^2\right)$ is always negative. Hence, both $\Delta(Q)^2$ and $\Delta(J)^2$ are bounded in the region $\frac{J}{A} < \frac{0.4}{8\pi}$. Thus, a perfect equilibrium is maintained between outgoing and incoming quanta for both charge and angular momentum in that region. However, equilibrium is lost for only angular momentum in the region $\frac{0.4}{8\pi} < \frac{J}{A} < \frac{1}{8\pi}$. The

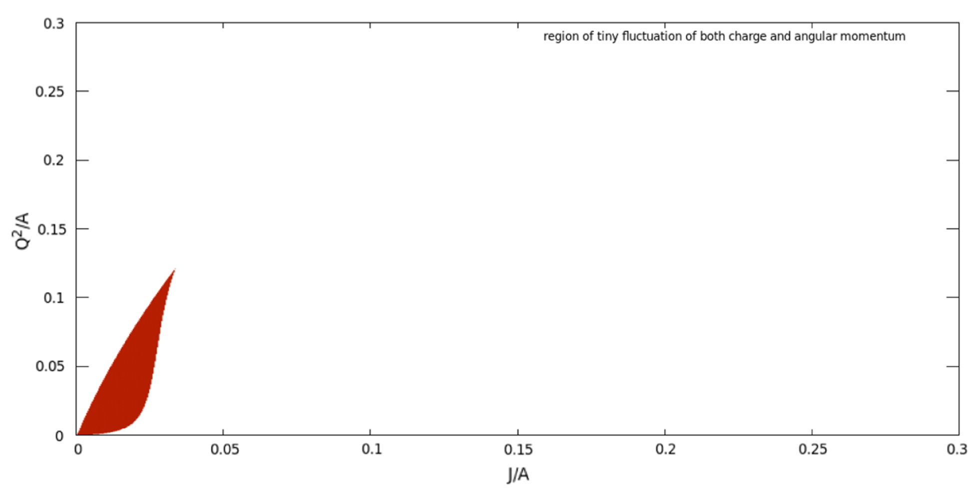

Fig. 4 Pictorial representation of region of bounded fluctuation for both angular momentum and charge

KSBH ultimately decays due to an unbounded area fluctuation, but we will see soon that the decay rate slows in certain regions of parameter space.

If the angular momentum (J) lies in the region $\frac{J}{A} < \frac{0.4}{8\pi}$, then J does not fluctuate much as its fluctuation is bounded there. However, the area (A) decreases as usual and causes $\frac{J}{A}$ to be greater than $\frac{0.4}{8\pi}$. Once this happens, large fluctuations of J begin, but positivity of the temperature restricts the ratio $\frac{J}{A}$ to be lower than $\frac{1}{8\pi}$ with decreasing area (A). Hence, J reduces to make the ratio $\frac{J}{A}$ smaller than $\frac{0.4}{8\pi}$. This process repeats. Hence, a KSBH reduces its angular momentum to satisfy its extremality bound during the Hawking decay, as the black hole gradually loses area and angular momentum, keeping the charge unchanged. As a result, it proceeds to transform into a black hole with charge only. This transformation is purely thermodynamic in nature. Therefore, we find the difference between a KS and a KN black holes in terms of their closeness to their end states.

That a KS black hole, unlike a KN black hole, hardly discharges throughout its life is important to note. One has to go back to the construction of the grand canonical partition to understand this. We, in this analysis, have assumed the mass of a rotating charged black hole to be a function of its area, angular momentum and charge. In any theory, these are good self-adjoint operators. Although mass is not a good primary operator, we can still represent it as a secondary operator in terms of other primary operators. Hence, we here consider fluctuations of area, charge and angular momentum only. In semiclassical analyses, one gets many constraints on the parameter space from the condition of avoiding a naked singularity. We, in a thermodynamic analysis, equivalently obtain similar constraints from the condition of avoiding absolute zero temperature. Semiclassically, a charged rotating KN black hole has been shown to lose its charge and angular momentum [23], just from the condition of various restrictions on the parameter space. We here also obtain similar results for KN black holes, just from the condition of various restrictions on the parameter space imposed by positivity of the temperature. However, this analysis is a bit interesting for a KS black hole. Positivity of the temperature does not put any bound on its electric charge. Close to the end state, this black hole loses almost all its angular momentum. The area also becomes comparable to the Planck area [24]. Hence, its mass is approximately given as $M^2 \approx \frac{Q^2}{2}$. This is very similar to stable, extremal black holes with magnetic monopoles. Of course, the last example is the outcome of a semiclassical analysis, where the mass of this black hole in the limiting case is given as $M^2 \approx P^2$, where P is the magnetic charge. We compare our thermodynamic analysis with well-known semiclassical analyses not to establish our analysis, but to show the simplicity, as well as superiority, of our analysis. B.carter, through his semi-classical analysis [25], showed that a charged black hole with initial mass on the order of 10^{15} kg does negligibly discharge throughout its life. This, if translated for a KS black hole, implies that a KS black hole almost does not discharge if its initial charge is roughly one mole of electrons. In fact, charged black holes with sufficient initial mass, under certain idealized conditions, have been shown semi-classically [26] not to discharge. This again supports our conclusion regarding the stability of an electric charge for decaying KS black holes.

4 Wuasi stable fluctuations and Hawking decay

We see from the above examples that bounded fluctuations exist for charged, rotating quasi-stable black holes in certain regions of their parameter space, like a stable AdSKNBH. However, they ultimately decay, with a query as to whether their quasi-stability can reduce their decay rate. That the calculation of the decay rate of any black hole still cannot be completed in any theory of quantum gravity is of note; Thus, we are really unable to calculate the lifetime of any black hole. Nevertheless, we, as you will see, can still obtain various pieces of interesting information regarding black hole decay just from the thermodynamic analysis. In fact, we will successfully answer the question that has been posed.

The decay of any black hole can approximately be explained by the Stefan–Boltzmann law as its profile for Hawking radiation approximately overlaps with that of a black body. Thus, its luminosity (L) varies with temperature(T) as $L \propto T^4$. As the temperature($\propto M_A$) is a function of its area, angular momentum and charge, the luminosity(L) is also a function of these parameters, i.e. $L = L(A, Q, J)$. Thus, the variation in luminosity can be expressed as

$$\Delta L \propto \left(M_{AA}\Delta A + M_{AJ}\Delta J + M_{AQ}\Delta Q\right). \tag{4}$$

We can express the mass of a black hole(M) as $M \equiv M(A, Q, J)$. This implies that $M_{AJ} = \frac{M_{AJ}^2}{2M} - \frac{M_A^2 M_J^2}{4M^3}$. That M_{AJ}^2 is negative for both KS and KN black holes can be easily checked. That M_A^2 and M_J^2 are positive for both of them is also true. Hence, M_{AJ} is negative for both of them, and a reduction in angular momentum ($\Delta J < 0$), which likely to be the case during the process of Hawking decay, enhances the decay rate.

Similarly, we can argue that M_{AQ} is negative for both KS and KN black holes. Because the ratio $\frac{Q^2}{A}$ for a KS black hole is not restricted, its charge may remain fixed at its equilibrium value. Hence, charge does not play any role in enhancing the decay rate of this black hole. The situation is different for a KN black hole. The ratio $\frac{Q^2}{A}$ is bounded; hence, the charge has to be reduced during the decay process to satisfy the non-extremality bound. This will, in turn, enhance the decay rate.

Now, M_{AA} is positive for a KS black hole in the region $\frac{0.4}{8\pi} < \frac{J}{A} < \frac{1}{8\pi}$. Thus, a decaying KS black hole reduces its decay rate as its area always decreases($\Delta A < 0$). Similarly, M_{AA} is positive for a KN black hole in the region $\frac{A^2}{96\pi^2} < \left(4J^2 + Q^4\right) < \frac{A^2}{16\pi^2}$. Thus, the decay rate decreases in this region, and we see that the presence of angular momentum for both KS and KN black holes enhances their lifetime in comparison with Schwarzschild black holes, supporting the semiclassical prediction [23]. Apart from these regions, both KN and KS black holes show their affinity to support Hawking decay. Hence, these black holes will gradually lose their angular momentum and charge(only for KN black holes).

Thus, we see that quasi-stable black holes try to resist the decay process, unlike an unstable AFSBH. However, they,unlike a stable AdSKNBH, ultimately decay under Hawking radiation.

5 Discussion

Charged, rotating, quasi stable black holes ultimately decay under Hawking radiation, like unstable black holes. However the angular momentum and the charge almost do not fluctuate in certain regions of their parameter space. These black holes try to resist the decay processes, somewhat similar to a stable black hole. Hence, they possess this dual property. However, the most fascinating thing is that quasi stable black holes transform from one kind to another during Hawking decay. This interesting property is solely a feature of quasi stable black holes. Both KNBH and KSBH have similar kinds of fluctuations during their Hawking decay, but the origins of these black holes are entirely different. Thus, some sort of differences is expected to be somewhere for them. We have unrevealed this in this paper. The difference is in their thermodynamic transformation. A KN black hole is likely to lose both its charge and angular momentum whereas a KS black hole is likely to lose its angular momentum only. One more fantastic issue deserves notice. We still do not know how to calculate the decay rate and the life time of any black hole from the perspective of Quantum Gravity, but we still can predict the leading order behavior of black holes during their decay. We can at least confirm whether the decay rate will increase or decrease at any point of parameter space. This is the novelty of this paper.

Obviously the end states of black holes will be fixed with some minimum area [24]. A KN black hole will end up as a neutral, non-rotating black hole whereas a KS black hole will end up as a non rotating, charged black hole. In fact, these remnants are potential candidates for dark matter, as well. Thus, the results described here may have relevance for dark matter physics [24].

Fluctuations of angular momentum and charge for both KNBH and KSBH behave like that of a stable AdSKNBH in some regions of parameter spaces. Now, an AdSBH is dual due to AdS/CFT correspondence to a strongly coupled gauge theory at finite temperature [27, 30]. Hence, strongly correlated condensed matter physics may be analyzed using the AdS/CFT correspondence. Thus, the results obtained here may have some impacts on condensed matter physics.

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