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Multi-echelon green supply chain model with random defectives, remanufacturing and rework under setup cost reduction and variable transportation cost

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Abstract. This article investigates a multi-echelon green supply chain system where a regular production process is integrated with a remanufacturing process in a single-setup-multi-delivery (SSMD) system under setup cost reduction. To reduce the setup cost, a discrete capital investment is introduced. A random number of the collected returned items are defectives which are uniformly distributed in the collected lot. The remanufacturing and manufacturing, both of the processes are imperfect. During the production run, they produce a random number of defectives that follow an exponential distribution. All these defectives are immediately reworked. Two models: Model-I and Model-II are developed in this regard. The transportation cost of the producer depends on the number of shipments which is a variable. The prime purpose of this study is to reduce the expected integrated total cost by optimizing the investment for setup cost reduction, common cycle length of all the buyers, total number of shipments, order lot size of each buyer, and delivery lot size of the producer. Numerical examples are provided to establish the models and the results show that Model-I can save more money than Model-II. Sensitivity analyses of the optimal solutions for both of the models are performed.

Keywords. Green supply chain; random defective; remanufacturing; rework; setup cost reduction; variable transportation cost.

1. Introduction

Green supply chain practice is a very useful, efficient and effective concept to carry on the environmental as well as ecological responsibility. To protect the environmental health, the government has imposed several rules and regulations on the manufacturing companies. The manufacturing companies have also adopted all possible measures to reduce environmental pollution and waste. But what to be done with the used and disposed items which are increasingly piled up and polluted the environment? After passing many decades, finally, they have got the best answer to the question that is reuse and remanufacturing.

Remanufacturing is one of the best activities of a green supply chain study to protect the environment from these increasing piles of wastes. Reusing the disposable items as a rich source of raw materials for remanufacturing is an intellectual idea to utilize the resources and save the environmental health. On one side, the concept of remanufacturing of used item helps to reduce environmental waste. On the other-hand, the rework of defectives which are generated during a manufacturing and/or a remanufacturing process is also a good strategy to reduce the production as well as environmental waste. It also helps to make the expected financial gain. The present study is motivated by ideas mentioned above.

Integrating a remanufacturing process with a regular manufacturing process needs a large setup cost. To reduce this setup cost as well as the total cost of the producer, an investment for setup cost reduction is a fruitful strategy. Although, many research works have been incorporated based on remanufacturing, integration of a remanufacturing process with a regular manufacturing process in a SSMD system under setup cost control together with the consideration of random defectives and rework in a multi-stage supply chain system has yet not found. The contribution of the study is to fill up this gap.

The article is split into eight sections. Section 1 gives the introduction of the study. Literature survey is given in Sect. 2. Section 3 contains the assumptions of the models. The model description and mathematical formulation are depicted in Sect. 4 and the solution procedure and

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theoretical results are stated in Sect. 5. Sections 6 and 7 illustrate the numerical examples and sensitivity analyses, respectively. The conclusion of the study with future extension is presented in Sect. 8.

2. Literature survey

The importance of green supply chain encourages several researchers to focus in this direction. Recently, Zhang and Liu [1] have investigated a three-echelon green supply chain model in which demand is influenced by the green degree of product while Saxena *et al* [2] have incorporated a two-level single-vendor single-buyer green supply chain system where trade credit financing is taken into consideration. A revenue-sharing contract in a green supply chain is discussed by Song and Gao [3]. Yang *et al* [4] have presented a credit policy for a green supply chain model with the consideration of capital constraints.

There exists quite a large number of literature that have studied inventory models with defectives or imperfect quality products. Khouja and Mehrez [5] have analyzed an economic production lot size (EPLS) model for imperfect quality items where they have considered rework of defectives. An economic order quantity (EOQ) model containing imperfect quality products is discussed by Salameh and Jaber [6] in which the imperfect quality products are sold in a single batch after a screening process. Chan *et al* [7] have integrated rework of defective items with lower pricing and rejection situation in an economic production quantity model while a random defective rate with rework and backlogging is taken into consideration by Chiu *et al* [8]. Cardenas-Barron [9] has modelled a multi-stage imperfect production system with a rework process whereas EOQ models with random defective products and exponential partial backlogging are presented by Das Roy *et al* [10, 11]. Supply chain models with reworkable items are incorporated by Pal *et al* [12] and Das Roy *et al* [13]. Wee *et al* [14] have optimized an unreliable economic production quantity (EPQ) model in which they have considered a non-synchronized screening process and rework while an unreliable production-inventory model is discussed by Das Roy and Sana [15] where the defectives are assumed to be sold with a free repair warranty offer. Sarkar [16] has proposed two different types of imperfect production systems: multi-stage single-cycle and multi-stage multi-cycle with rework consideration. Besides these, inventory models with imperfect/defective products are also discussed in several research articles [17–21].

Natural resources are limited. Recently, remanufacturing/recycling of used product is a growing interest of the manufacturing industries. The company may collect the used products from the end customers directly at less value and may use these used products as raw materials of the remanufacturing system. Last few decades, many authors have included remanufacturing in their study. In 1967, Schrady [22] introduced recovery and remanufacturing in a traditional economic order quantity (EOQ) model and determined optimal lot size. Kiesmuller [23] has discussed the effect of lead time in a stochastic remanufacturing system while push and pull strategies together with the operations of remanufacturing and disposal are introduced by van der Laan and Teunter [24]. Rubio and Corominas [25] have presented an optimal manufacturing as well as remanufacturing strategy in the environment of a lean production process whereas a closed-loop supply chain inventory model with the consideration of remanufacturing is incorporated by Chung *et al* [26]. A remanufacturing system with uncertain demand and quality is considered by Mukhopadhyay and Ma [27]. Hsueh [28] has investigated a manufacturing or remanufacturing system with the different phases of a product life cycle and concluded that the different inventory policies have to be adopted in different phases of the product life-cycle. An inventory model with remanufacturing, waste disposal together with complete and partial backlogging is framed by Hasanov *et al* [29] where they have shown that the outcomes of remanufacturing cannot satisfy the needs of consumers due to lack of sufficient storage capacity. Saxena *et al* [2] have investigated a green supply chain where they have integrated a production process of new items with a remanufacturing process under trade credit policy and variable transportation cost, whereas an inventory model for uncertain consumption rate and acquisition quantity including remanufacturing process is studied by Liao and Deng [30]. Saxena *et al* [31] have investigated a supply chain model with a manufacturing/remanufacturing process where they have discussed two approaches of replenishment cycles regarding waste control.

Porteus [32] was considered to be the first researcher who has introduced setup cost reduction in an inventory model. After Porteus [32], many authors [33, 34] have taken setup cost control into consideration. Uthayakumar and Priyan [35] have introduced setup cost reduction together with lead time reduction in a supply chain model while a combined effect of lead time, setup and quality control with trade credit and transportation discount is studied by Kim and Sarkar [36]. Das Roy [37] has introduced setup cost reduction with exponential lead time crashing cost in a two-level supply chain inventory model while production rate and lot size-dependent lead time reduction strategies in a two-stage supply chain model with setup cost reduction is presented by Das Roy and Sana [38].

The proposed article studies a multi-stage green supply chain inventory model with a single supplier, a single manufacturer's warehouse, a single producer, and multiple buyers. The used returned items are collected in the manufacturer's warehouse. This collection contains a random number of defectives. During a screening process, these defectives are separated. The percentage of such defective items is uniformly distributed in the collected lot.

The production process starts with remanufacturing by using the collected used items. At the same time, the producer places an order of raw materials for the regular production which are reached and stored at the manufacturer's warehouse at the end of the remanufacturing cycle. In reality, every machinery system is unreliable to produce 100% perfect quality products. Consequently, a certain percentage of products are imperfect which are reworked to make them as new as the original quality products. In most studies, remanufacturing system is considered as a 100% reliable production system that generates only perfect quality items. In realistic points of view, if a forward manufacturing system may produce defective products, then a remanufacturing process may also generate defective items which may be in very small percentage as both the systems deal with machines. The present paper assumes that during remanufacturing as well as regular production time, a random number of defectives are generated which are immediately reworked. The remanufactured and newly produced items, both are of the same quality. The producer stores them in the same place and meets the demand of all buyers by them. The producer invests an additional cost to reduce setup cost. The transportation cost of the produced depends on the number of shipments. A general flow of inventory in the supply chain is shown in figure 1. The objective of this study is to serve the purpose of reducing environmental waste and expected integrated total cost.

3. Assumptions

The assumptions used to frame the proposed multi-echelon green supply chain model are as follows.

1. This study considers a multi-stage green supply chain system consists of a supplier, a manufacturer's warehouse, a producer and multiple buyers to trade a single type of product.
2. In this model, remanufacturing/recycling the stock of used items collected during the previous cycle is attempted first and then fresh production process is taken into consideration. So, the manufacturing cycle begins with remanufacturing and the producer starts remanufacturing by using the collected returned items as raw material. The manufacturer places an order for raw material at the same time because he purchases raw materials from a distant supplier as the quality of the raw materials of that supplier is good and the producer does not like to face any delay or shortage of raw materials during the production of new items.
3. A random percentage (ξ) of the returned items are defective which follows a uniform distribution with probability density function

$$f(\xi) = \frac{1}{\xi_2 - \xi_1}, \xi_1 \leq \xi \leq \xi_2$$
$$= 0, \text{elsewhere}.$$

4. The remanufacturing and manufacturing processes are imperfect. They produce β % and γ% of random defectives respectively which follow the exponential distribution with the following probability density functions.

$$g_1(\beta) = \delta_1 e^{-\delta_1 \beta}, \beta \geq 0$$
$$= 0, \text{elsewhere}$$

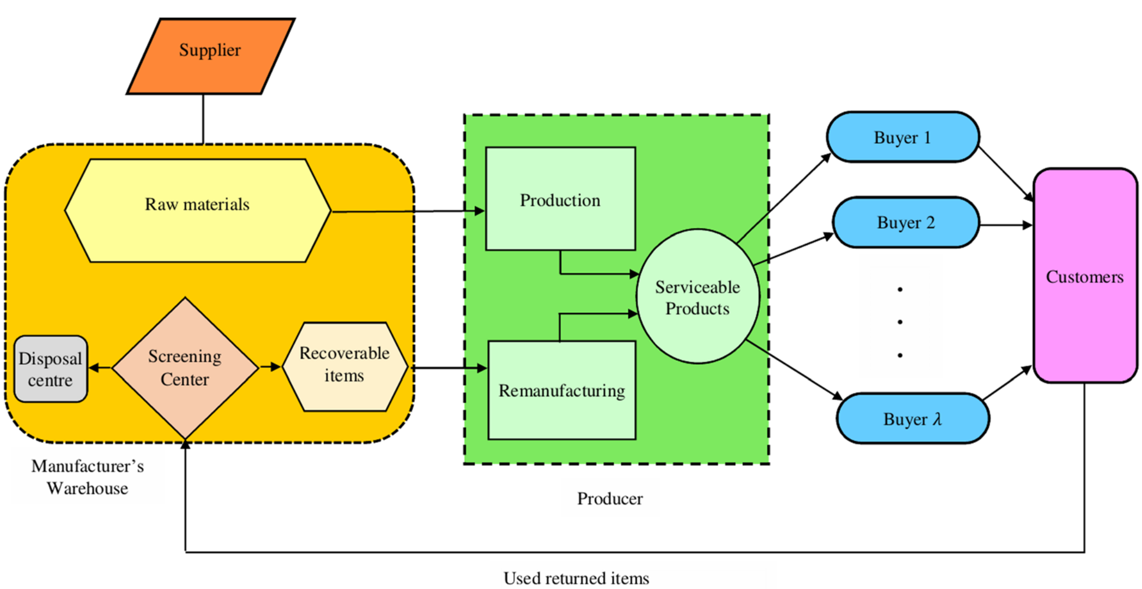

Figure 1. Flow of inventory in a multi-echelon green supply chain.

$$\text{and } g_2(\gamma) = \delta_2 e^{-\delta_2\gamma}, \gamma \geq 0$$
$$= 0, \text{elsewhere.}$$

These defective products are immediately reworked and restored in the producer's storage.

5. A discrete investment cost is introduced to reduce the producer's setup cost.
6. In each production cycle, the producer produces $n\sum_{i=1}^{\lambda} q_i$ units and ships them to the buyers into n shipments. Out of which, u number of the deliveries are done by the re-manufactured products and v number of the deliveries are done by the newly manufactured items.
7. Producer's transportation cost is variable. It depends on the number of deliveries.
8. The cycle lengths of all the buyers are equal.

4. Model description and mathematical formulation

The ith buyer orders nq_i, $i = 1, 2, \ldots, \lambda$ units to the producer and the producer delivers the order of ith buyer into n shipments [21], each of order lot size q_i. Each delivery lot reaches to buyer premises when the previous lot is completely depleted. The producer meets the order of each buyer with u number of deliveries of the remanufactured items and the v number of deliveries of the newly produced items. The collection of used returned items and the raw materials for new production are stored in a manufacturer's warehouse. A random percent ξ of the collected used return items are defectives which are not at the condition of reuse as raw materials for remanufacturing. These non-recoverable defective items are separated during the screening process in the manufacturer's warehouse. The remaining $(1-\xi)\%$ of the collected used return items are reused in the remanufacturing process.

The production cycle begins with remanufacturing at a rate K_r by using the accumulated reusable returned items and produces uDT units where $D = \sum_{i=1}^{\lambda} d_i$. Therefore,

$$(1-\xi)RnT = K_r.\frac{uDT}{K_r}$$

$$\text{or,}\quad u = \frac{(1-\xi)\text{Rn}}{\text{D}} \tag{1}$$

$$\text{and}\quad v = n - u = n\left[1 - \frac{(1-\xi)\text{R}}{\text{D}}\right]. \tag{2}$$

At the same time, the producer orders for raw material to a distant supplier as his quality of raw materials are good. To avoid shortage and delay in the production of new items, the producer orders raw materials at the start of the remanufacturing process. These raw materials are reached and also stored at the manufacture's warehouse within the remanufacturing cycle. At the end of the remanufacturing cycle, the producer begins to produce new items at a rate K_m and ships them into v number of shipments to the buyers. The remanufacturing and manufacturing, both of the processes are imperfect. Let $\beta\%$ and $\gamma\%$ of random defectives are generated which are immediately reworked. The remanufactured, new produced, and reworked items are of the same quality. They are stored in the same storage at the producer's premises for delivery. The change of inventory levels of the supplier, manufacturer's warehouse, producer and buyers are shown in figure 2.

Two models: Model-I with setup cost reduction and Model -II without setup cost reduction are discussed in the following subsections.

4.1 *Model-I: with setup cost reduction*

In this model, a discrete capital investment cost is considered to reduce the setup cost of the entire manufacturing system. At first, we will calculate the individual costs of the supplier, manufacturer's warehouse, producer and buyers as follows.

4.1.1 *Supplier's individual cost* The supplier of raw materials gets an order to supply vDT amounts of raw materials within the time period $\left[0, \frac{DT}{K_r} + (u-1)T\right]$, where D is the demand rate of the manufacturer's warehouse as well as the producer.

$$\text{Therefore,}\quad D\left[\frac{DT}{K_r} + (u-1)T\right] = vDT$$

$$\text{which implies,}\quad \left[\frac{DT}{K_r} + (u-1)T\right] = vT \tag{3}$$

The cycle length of the supplier is nT. Therefore, the ordering cost and transportation cost of the supplier per unit time are $\frac{A_s}{nT}$ and $\frac{F_s}{nT}$ respectively.

Purchase cost The supplier purchases vDT from some other supplier at a rate p_s. So, the purchase cost of the supplier per unit time is $= \frac{p_s vD}{n} = p_s[D - (1-\xi)R]$.

Holding cost The supplier holds vDT or $D\left[\frac{DT}{K_r} + (u-1)T\right]$ units for the time span $\left[0, \frac{DT}{K_r} + (u-1)T\right]$ (see figure 2). Using the relations (3) and (2), the holding cost of the supplier per unit time becomes

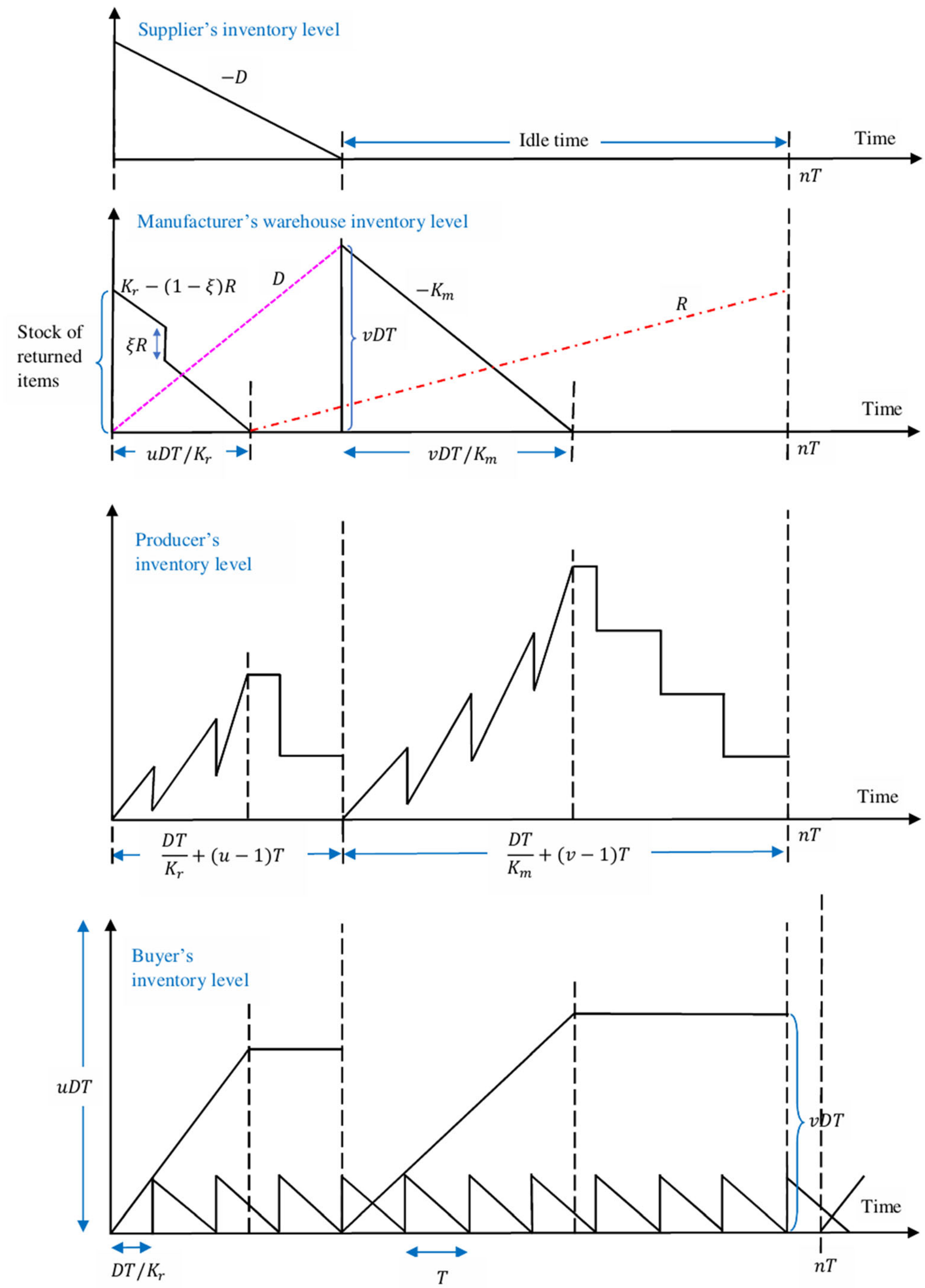

Figure 2. Inventory versus time.

$$=\frac{h_s}{nT}\cdot\frac{1}{2}.D\left[\frac{DT}{K_r}+(u-1)T\right]^2$$

$$=\frac{h_sDv^2T}{2n}=\frac{h_sDTn}{2}\left[1-\frac{(1-\xi)R}{D}\right]^2.$$

Idle cost After delivering the order, the supplier remains idle for the rest of the time $\left[nT-\left\{\frac{DT}{K_r}+(u-1)T\right\}\right]$. So, the idle cost of the supplier per unit time is

$$=\frac{I_s}{nT}\left[nT-\left\{\frac{DT}{K_r}+(u-1)T\right\}\right]=\frac{I_s}{n}(n-v)$$
$$=\frac{I_s(1-\xi)R}{D}.$$

The expected total cost of the supplier per unit time is

$$ETS=E[\text{Ordering cost}+\text{Purchase cost}+\text{Holding cost}+\text{Transportation cost}+\text{Idle cost}]$$

$$= \frac{A_s}{nT} + p_s[D - (1 - E[\xi])R] + \frac{h_s DTn}{2} E\left[1 - \frac{(1-\xi)R}{D}\right]^2 + \frac{F_s}{nT} + \frac{I_s(1 - E[\xi])R}{D}.$$

4.1.2 *Manufacturer's warehouse individual cost* In the manufacturer warehouse, both used returned items for remanufacturing and raw materials for new production are stored. The relevant costs at the manufacturer's warehouse per unit time are as follows.

Ordering cost The ordering cost of raw materials $= \frac{A_w}{nT}$.

Purchase cost The purchase cost of raw materials for manufacturing new products is

$$= \frac{p_w vD}{n} = p_w[D - (1-\xi)R].$$

Accusation cost The manufacturer collects used items from customers for remanufacturing. The return rate of such items is R. So, the accusation cost for returned items is $= \frac{C_a RnT}{nT} = C_a R$.

Screening Cost The stock of returned items is screened at a rate S_r and the screening cost $= S_C R$.

Holding cost The holding cost is the sum of the holding cost of raw materials for regular production and the holding cost of the collected returned items. Now, the inventory of raw materials (see figure 2) includes the accumulation of raw materials during $\left[0, \frac{DT}{K_r} + (u-1)T\right]$ and inventory of raw materials during the regular production run time. Therefore, the inventory of raw materials is

$$= \frac{1}{2}.D\left[\frac{DT}{K_r} + (u-1)T\right]^2 + \frac{1}{2}.K_m\left[\frac{vDT}{K_m}\right]^2$$

$$= \frac{1}{2}Dv^2T^2\left(1 + \frac{D}{K_m}\right)$$

$$= \frac{1}{2}DT^2n^2\left[1 - \frac{(1-\xi)R}{D}\right]^2\left(1 + \frac{D}{K_m}\right).$$

The inventory of used returned items = inventory of recoverable items used in remanufacturing + inventory of non-recoverable defective items

$$= \frac{T^2}{2}\left[\frac{(K_r - (1-\xi)R)u^2D^2}{K_r{}^2} + \frac{2\xi R^2 n^2}{S_r} + R\left(n - \frac{uD}{K_r}\right)^2\right]$$

$$= \frac{T^2n^2}{2}\left[\frac{(K_r - (1-\xi)R)(1-\xi)^2R^2}{K_r{}^2} + \frac{2\xi R^2}{S_r} + R\left(1 - \frac{(1-\xi)R}{K_r}\right)^2\right].$$

Therefore, the holding cost per unit time is

$$= \frac{h_w Tn}{2}\left[D\left[1 - \frac{(1-\xi)R}{D}\right]^2\left(1 + \frac{D}{K_m}\right) + \frac{(K_r - (1-\xi)R)(1-\xi)^2R^2}{K_r{}^2} + \frac{2\xi R^2}{S_r} + R\left(1 - \frac{(1-\xi)R}{K_r}\right)^2\right].$$

Disposal cost The disposal cost of defectives is $= C_d\xi R$.

Transportation cost The transportation cost from the manufacturer's warehouse to the production premises is

$$= \frac{F_w}{nT}.$$

The expected total cost of the manufacturer's warehouse per unit time is

$$ETW = E[\text{Ordering cost} + \text{Purchase cost} + \text{Accusation cost} + \text{Screening cost} + \text{Holding cost} + \text{Disposal cost} + \text{Transportation cost}]$$

$$= \frac{A_w}{nT} + p_w[D - (1 - E[\xi])R] + C_a R + S_C R + \frac{h_w Tn}{2}\left[DE\left[1 - \frac{(1-\xi)R}{D}\right]^2\left(1 + \frac{D}{K_m}\right) + \frac{(K_r - (1 - E[\xi])R)E[(1-\xi)^2]R^2}{K_r{}^2} + \frac{2E[\xi]R^2}{S_r} + RE\left[1 - \frac{(1-\xi)R}{K_r}\right]^2\right] + C_d E[\xi]R + \frac{F_w}{nT}.$$

4.1.3 *Producer's individual cost* The producer produces $nQ = n\sum_{i=1}^{\lambda} q_i = n\sum_{i=1}^{\lambda} d_i T = nDT$ units, where $n = u + v$. Among them, uDT units are remanufactured and vDT units are newly produced. The inventory level of the producer is shown in figure 2 and his relevant costs per unit time are as follows.

Setup cost The producer invests an additional cost I_m to reduce the setup cost and hence the new setup cost becomes

$$A_m(I_m) = \frac{A_m^0 e^{-\alpha I_m}}{nT},$$

where A_m^0 is an initial setup cost and α is a fixed parameter.

Investment cost for setup cost reduction The investment for setup cost reduction $= \frac{I_m}{nT}$.

Manufacturing cost The remanufacturing and regular production costs are $C_u uDT$ and $C_v vDT$ respectively. Therefore, the total manufacturing cost per unit time is

$$= \frac{D}{n}(uC_u + vC_v) = C_u(1-\xi)R + C_v[D - (1-\xi)R].$$

Holding cost The holding cost of the producer is the sum of the holding cost of the remanufactured products and newly produced items. Now, the inventory of remanufactured products (see figure 2) is

$$= \left\{\frac{DT}{K_r} + (u-1)T\right\}uDT - \frac{uDT}{K_m}\left(\frac{uDT}{2}\right) - \{T + 2T + \ldots + (u-1)T\}DT$$

$$= \frac{uDT^2}{2}\left\{u\left(1 - \frac{D}{K_r}\right) - 1 + \frac{2D}{K_r}\right\}$$

and the inventory of newly produced items is

$$= \frac{vDT^2}{2}\left\{v\left(1 - \frac{D}{K_m}\right) - 1 + \frac{2D}{K_m}\right\}.$$

Therefore, the holding cost of the producer per unit time is

$$= \frac{h_m DT}{2n}\left[u\left\{u\left(1 - \frac{D}{K_r}\right) - 1 + \frac{2D}{K_r}\right\} + v\left\{v\left(1 - \frac{D}{K_m}\right) - 1 + \frac{2D}{K_m}\right\}\right]$$

$$= \frac{h_m T}{2}\left[n\left\{\frac{(1-\xi)^2R^2}{D}\left(1 - \frac{D}{K_r}\right) + D\left[1 - \frac{(1-\xi)R}{D}\right]^2\left(1 - \frac{D}{K_m}\right)\right\} + 2DR(1-\xi)\left(\frac{1}{K_r} - \frac{1}{K_m}\right) + D\left(\frac{2D}{K_m} - 1\right)\right].$$

Rework cost During the remanufacturing and manufacturing process, $\beta\%$ and γ % of defectives are respectively produced together with the perfect quality items. These defectives are immediately reworked and the rework cost is

$$= \frac{C_r}{nT}(\beta uDT + \gamma vDT)$$

$$= C_r[\beta(1-\xi)R + \gamma[D - (1-\xi)R]].$$

Transportation cost The producer transports the whole order into n shipments and the transportation cost for each shipment is F_m. Therefore, the transportation cost of the producer per unit time is

$$= \frac{nF_m}{nT}.$$

The expected total cost of the producer per unit time is

$$ETP = E[\text{Setup cost} + \text{Investment for setup cost reduction} + \text{Manufacturing cost} + \text{Holding cost} + \text{Rework cost} + \text{Transportation cost}]$$

$$= \frac{A_m^0 e^{-\alpha I_m}}{nT} + \frac{I_m}{nT} + C_u(1 - E[\xi])R + C_v[D - (1 - E[\xi])R]$$

$$+ \frac{h_m T}{2}\left[n\left\{\frac{E[(1-\xi)^2]R^2}{D}\left(1 - \frac{D}{K_r}\right) + DE\left[1 - \frac{(1-\xi)R}{D}\right]^2\left(1 - \frac{D}{K_m}\right)\right\} + 2DR(1 - E[\xi])\left(\frac{1}{K_r} - \frac{1}{K_m}\right) + D\left(\frac{2D}{K_m} - 1\right)\right]$$

$$+ C_r[E[\beta](1 - E[\xi])R + E[\gamma][D - (1 - E[\xi])R]] + \frac{nF_m}{nT}.$$

4.1.4 *Buyers' individual cost* The relevant costs of the ith buyer, $i = 1, 2, \ldots, \lambda$ per unit time are as follows.

Ordering cost The ordering cost of the ith buyer $= \frac{A_{bi}}{T}$.

Purchase cost The purchase cost of an item is same for all the buyers as they have purchased the same product from the same producer. So, the purchase cost of the ith buyer $= p_b d_i$.

Holding cost

$$\text{The holding cost of the } i\text{th buyer} = \frac{h_{bi} d_i T}{2}.$$

The expected total cost of the ith buyer per unit time is

$$ETB_i = E[\text{Ordering cost} + \text{Purchase cost} + \text{Holding cost}].$$

Therefore, the expected total cost of all the buyers per unit time is

$$ETB = \sum_{i=1}^{\lambda} ETB_i = \frac{1}{T}\sum_{i=1}^{\lambda} A_{bi} + p_b D + \frac{T}{2}\sum_{i=1}^{\lambda} h_{bi} d_i.$$

4.1.5 *Integrated total cost* The expected integrated total cost of the entire supply chain system is the sum of the expected total cost of the supplier, manufacture's warehouse, producer, and all the buyers which can be written as

$$ETIC(I_m, T, n) = ETS + ETW + ETP + ETB$$

$$= \frac{1}{nT}\left[J_0 + A_m^0 e^{-\alpha I_m} + I_m + n\left(F_m + \sum_{i=1}^{\lambda} A_{bi}\right)\right] + \frac{T}{2}\left[n(h_s J_3 + h_w J_4 + h_m J_5) + h_m J_6 + \sum_{i=1}^{\lambda} h_{bi} d_i\right] + J_7, \tag{4}$$

where $J_0 = A_s + A_w + F_s + F_w$, $J_1 = 1 - E[\xi] > 0$,

$$J_2 = E\left[(1-\xi)^2\right] = J_1{}^2 + Var(\xi)[\text{see Appendix A}],$$

$$J_3 = D\left(1 - \frac{2J_1 R}{D} + \frac{J_2 R^2}{D^2}\right)$$

$$J_4 = D\left(1-\frac{2J_1R}{D}+\frac{J_2R^2}{D^2}\right)\left(1+\frac{D}{K_m}\right)+\frac{(K_r-J_1R)J_2R^2}{K_r{}^2} + \frac{2E[\xi]R^2}{S_r}+R\left(1-\frac{2J_1R}{K_r}+\frac{J_2R^2}{K_r{}^2}\right)$$

$$J_5 = \frac{J_2R^2}{D}\left(1-\frac{D}{K_r}\right)+D\left(1-\frac{2J_1R}{D}+\frac{J_2R^2}{D^2}\right)\left(1-\frac{D}{K_m}\right)$$

$$J_6 = 2DRJ_1\left(\frac{1}{K_r}-\frac{1}{K_m}\right)+D\left(\frac{2D}{K_m}-1\right)$$

$$J_7 = (D-J_1R)(p_s+p_w+C_v+C_rE[\gamma]) + R(C_a+S_c+C_dE[\xi]) + J_1R\left(C_u+C_a+C_rE[\beta]+\frac{I_s}{D}\right).$$

4.2 *Model-II: without setup cost reduction*

In this model, a fixed setup cost is considered. Thus, the expected integrated total cost for this model becomes

$$\widetilde{ETIC}(T,n) = \frac{1}{nT}\left[J_0+A_m^0+n\left(F_m+\sum_{i=1}^{\lambda}A_{bi}\right)\right] + \frac{T}{2}\left[n(h_sJ_3+h_wJ_4+h_mJ_5)+h_mJ_6+\sum_{i=1}^{\lambda}h_{bi}d_i\right]+J_7, \tag{5}$$

where J_0, J_3, J_4, J_5, J_6 and J_7 are same as stated above.

5. Solution procedure and theoretical results

Differentiating equation (4) with respect to I_m, T and n as follows.

$$\frac{\partial ETIC}{\partial I_m} = \frac{1}{nT}\left(-\alpha A_m^0 e^{-\alpha I_m}+1\right)$$

$$\frac{\partial ETIC}{\partial T} = -\frac{1}{nT^2}\left[J_0+A_m^0e^{-\alpha I_m}+I_m+n\left(F_m+\sum_{i=1}^{\lambda}A_{bi}\right)\right] + \frac{1}{2}\left[n(h_sJ_3+h_wJ_4+h_mJ_5)+h_mJ_6+\sum_{i=1}^{\lambda}h_{bi}d_i\right]$$

$$\frac{\partial ETIC}{\partial n} = -\frac{1}{n^2T}\left(J_0+A_m^0e^{-\alpha I_m}+I_m\right) + \frac{T}{2}(h_sJ_3+h_wJ_4+h_mJ_5)$$

The optimal solutions are obtained by equating $\frac{\partial ETIC}{\partial I_m}$, $\frac{\partial ETIC}{\partial T}$, and $\frac{\partial ETIC}{\partial n}$ equals to zero, which gives

$$I_m{}^* = \frac{1}{\alpha}\ln(\alpha A_m^0) \tag{6}$$

$$T^* = \sqrt{\frac{2\left[J_0+A_m^0e^{-\alpha I_m}+I_m+n\left(F_m+\sum_{i=1}^{\lambda}A_{bi}\right)\right]}{n\left[n(h_sJ_3+h_wJ_4+h_mJ_5)+h_mJ_6+\sum_{i=1}^{\lambda}h_{bi}d_i\right]}} \tag{7}$$

$$n^* = \frac{1}{T}\sqrt{\frac{2\left(J_0+A_m^0e^{-\alpha I_m}+I_m\right)}{h_sJ_3+h_wJ_4+h_mJ_5}} \tag{8}$$

Theorem 1 *The expected integrated total cost $ETIC(I_m,T,n)$ for Model-I has a global minimum at the optimal solution $(I_m{}^*,T^*,n^*)$.*

Proof See “Appendix B”.

Theorem 2 *The optimum expected integrated total cost $\widetilde{ETIC}(T^*,n^*)$ for Model-II is*

$$\widetilde{ETIC}(T^*,n^*) = \frac{1}{n^*T^*}\left[J_0+A_m^0+n^*\left(F_m+\sum_{i=1}^{\lambda}A_{bi}\right)\right] + \frac{T^*}{2}\left[n^*(h_sJ_3+h_wJ_4+h_mJ_5)+h_mJ_6+\sum_{i=1}^{\lambda}h_{bi}d_i\right]+J_7 \tag{9}$$

and the optimal solutions are

$$T^* = \sqrt{\frac{2\left[J_0+A_m^0+n^*\left(F_m+\sum_{i=1}^{\lambda}A_{bi}\right)\right]}{n^*\left[n^*(h_sJ_3+h_wJ_4+h_mJ_5)+h_mJ_6+\sum_{i=1}^{\lambda}h_{bi}d_i\right]}} \tag{10}$$

$$n^* = \frac{1}{T^*}\sqrt{\frac{2\left(J_0+A_m^0\right)}{h_sJ_3+h_wJ_4+h_mJ_5}} \tag{11}$$

Proof By substituting, $I_m = 0$ in Eqs. (4), (7) and (8), the expressions stated in Equ. (9), (10), and (11) are obtained. Hence the proof. □

Theorem 3 *The expected integrated total cost $\widetilde{ETIC}(T,n)$ for Model-II has a global minimum at the optimal solution (T^*,n^*).*

Proof See “Appendix C”.

6. Numerical examples

The theoretical outcomes are established with the help of the following numerical examples whose most of the data are taken from Saxena *et al* [2].

Example 1. Suppose the random defective items in the accumulated returned items follow a uniform distribution with the following probability density function:

$$f(\xi) = 10, \text{when } 0 \leq \xi \leq 0.1$$
$$= 0, \text{otherwise.}$$

Also, the random defective products produced during the remanufacturing and manufacturing processes, follow exponential distributions with the following probability density functions respectively:

$$g_1(\beta) = 10e^{-10\beta}, \beta \geq 0$$
$$= 0, \text{elsewhere}$$
$$\text{and } g_2(\gamma) = 15e^{-15\gamma}, \gamma \geq 0$$
$$= 0, \text{elsewhere.}$$

The other parameter values are: $\lambda = 3, d_1 = 1000$ units/time, $d_2 = 1100$ units/time, $d_3 = 900$ units/time, $D = 3000$ units/time, $K_m = 5000$ units/time, $K_r = 4000$ units/time, $R = 880$ units/time, $A_s = \$400$/order, $A_w = \$760$/order, $A_m^0 = \$2000$/setup, $A_{b1} = \$23$/order, $A_{b2} = \$24$/order, $A_{b3} = \$22$/order, $h_s = \$0.1$/unit/time, $h_w = \$0.15$/unit/time, $h_m = \$0.2$/unit/time, $h_{b1} = \$1.2$/unit/time, $h_{b2} = \$1.5$/unit/time, $h_{b3} = \$1$/unit/time, $p_s = \$1$/unit, $p_w = \$3$/unit, $p_b = \$10$/unit, $C_a = \$1$/unit,, $C_u = \$2.5$/unit, $C_v = \$4$/unit, $S_r = 60,000$ units/unit time, $S_c = \$0.2$/unit, $C_d = \$0.1$/unit, $C_r = \$0.1$/unit, $F_s = \$15$/order, $F_w = \$20$/order, $F_m = \$36$/shipment, $\alpha = 0.0032$, and $I_s = \$50$/time. The optimal results for Model -I are recorded in table 1.

From table 1, it is observed that the optimal solutions are: ${I_m}^* = \$580.09$/production run, $T^* = 0.23145$ unit, $n^* = 10, u^* = 2.79, v^* = 7.21$, ${q_1}^* = 231$ units, ${q_2}^* = 254$ units, ${q_3}^* = 208$ units, $Q^* = 693$ units and the expected integrated total cost $ETIC^* = \$53200.50$. The graphical representations in figures 3, 4, and 5 show the convexity of the expected integrated total cost function for Example 1 with respect to the decision variables.

Example 2. All the parameter values are same as Example 1. The optimal solutions for Model –II are recorded in table 2.

The optimal solutions for Model -II are: $T^* = 0.23145$ unit, $n^* = 12, u^* = 3.34$, $v^* = 8.66$, ${q_1}^* = 231$ units, ${q_2}^* = 254$ units, ${q_3}^* = 208$ units, $Q^* = 693$ units and the expected integrated total cost $\widetilde{ETIC}^* = \$53625.90$. The graphical representation in figure 6 shows the convexity of the expected integrated total cost for Example 2.

6.1 *Comparison*

A comparison between Model-I and Model-II with the help of numerical results are stated in table 3.

6.2 *Discussion*

- From table 3, it is noted that the setup cost for Model-I is lower than Model-II. An introduction of additional investment for setup cost reduction significantly reduces the setup cost per cycle of the producer. It is approximately 84.38%.
- The transportation cost of the producer increases with the increase in the number of shipments. The number of shipments for Model-I is lower than Model-II (see table 3). As a result, the transportation cost of the producer is reduced approximately 16.67% in Model-I than Model-II.
- Clearly, $ETIC^* < \widetilde{ETIC}^*$ i.e. the expected integrated total cost for Model-I is less than Model -II. The joint reduction in the setup cost and transportation cost of the producer results approximately 0.8% reduction in the expected integrated total cost in Model-I.

7. Sensitivity analyses

The values of the parameters K_m, K_r, R, A_s, A_w, A_m^0, h_s, h_w, h_m, F_s, F_w, F_m, and α are changed by − 50%, − 25%, + 25% and + 50% while keeping the other parameter values unchanged to study the effect of these changes on the optimal solutions of Example 1 and Example 2.

The observations of the sensitivity analyses recorded in different tables are as follows.

- The expected integrated total cost ($ETIC^*$) and number of shipments (n^*) of Example 1 decrease with an increase in regular production rate (K_m) but the investment for setup cost reduction (${I_m}^*$) remains unchanged and the common cycle length of all the buyers (T^*) is increased (see table 4). On the other-hand, n^* decreases but T^* and $\widetilde{ETIC}^*$ of Example 2 increase with an increase in K_m.
- While the remanufacturing rate (K_r) increases, the expected integrated total cost and T^* for both Example 1 and 2 increase but n^* is decreased (see table 5) and ${I_m}^*$ remains unaltered.
- If the returned rate of used items (R) increases, then $ETIC^*$, $\widetilde{ETIC}^*$, and T^* for Example 1 and Example 2 decrease (see table 6) but n^* increases and ${I_m}^*$ stays unchanged.
- From tables 7 and 8, it is noted that, when the ordering costs of the supplier (A_s) and manufacturer's

Table 1. The optimal solutions for Model –I.

$I_m{}^*$	T^*	n^*	u^*	v^*	$q_1{}^*$	$q_2{}^*$	$q_3{}^*$	Q^*	$ETIC^*$
580.09	0.23145	10	2.79	7.21	231	254	208	693	53200.50

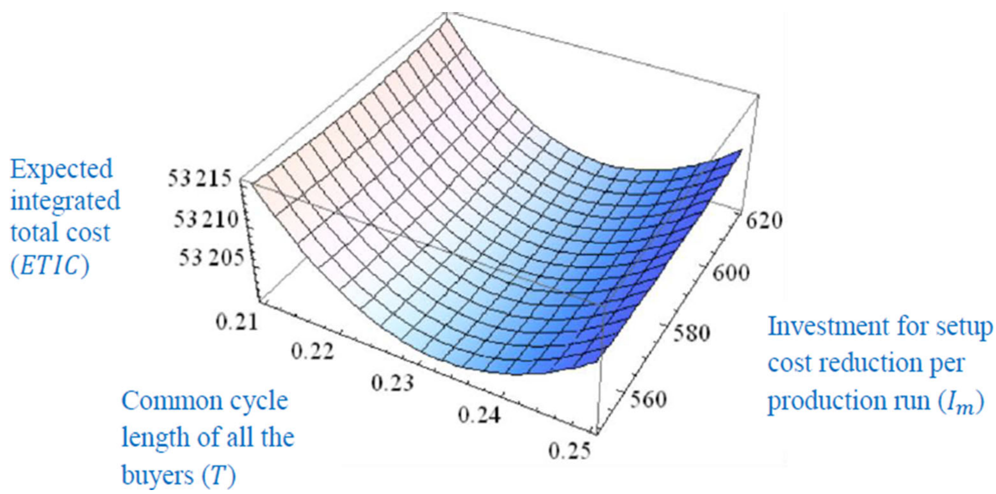

Figure 3. Graphical representation of the expected integrated total cost (*ETIC*) versus common cycle length of all the buyers (*T*) and investment of setup cost reduction per production run (I_m) for Example 1, when *n* is fixed.

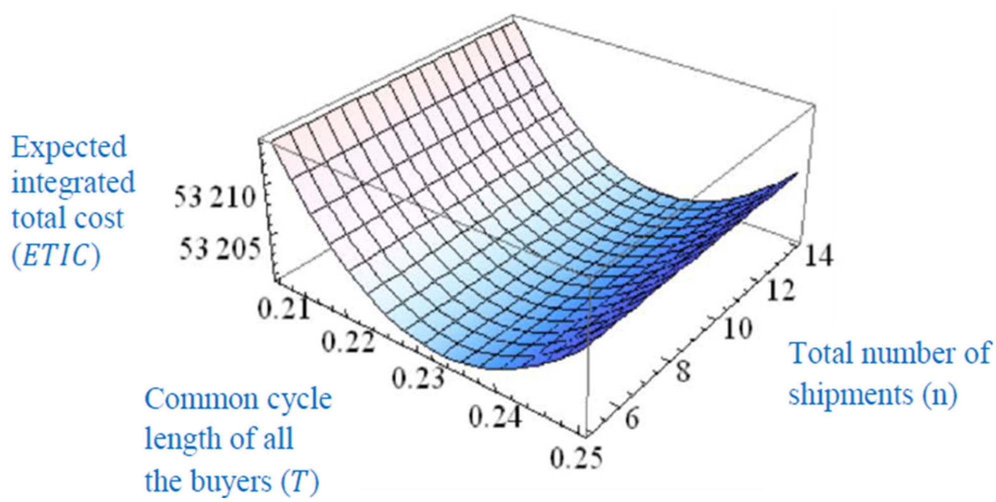

Figure 4. Graphical representation of the expected integrated total cost (*ETIC*) versus common cycle length of all the buyers (*T*) and total number of shipments (*n*) for Example 1, when I_m is fixed.

warehouse (A_w) are increased, $I_m{}^*$ and T^* remain unaltered but n^* decreases while $ETIC^*$ and $\widetilde{ETIC}^*$ are increased.

- Table 9 shows that T^* remains unchanged with the increase of initial setup cost (A_m^0) but $I_m{}^*$, n^*, $ETIC^*$ and $\widetilde{ETIC}^*$ are increased with an increase in A_m^0.
- When the holding cost of the supplier (h_s) and the manufacturer's warehouse (h_w) increase then $I_m{}^*$ and T^* remain unchanged but n^* decreases whereas $ETIC^*$ and $\widetilde{ETIC}^*$ for Examples 1 and 2 increase (see tables 10 and 11).
- The common cycle length of all the buyers (T^*) decreases while the expected integrated total cost for Examples 1 and 2 are increased with an increase in the holding cost (h_m) of the producer (see table 12). $I_m{}^*$ remains unchanged as it is independent of h_m but the

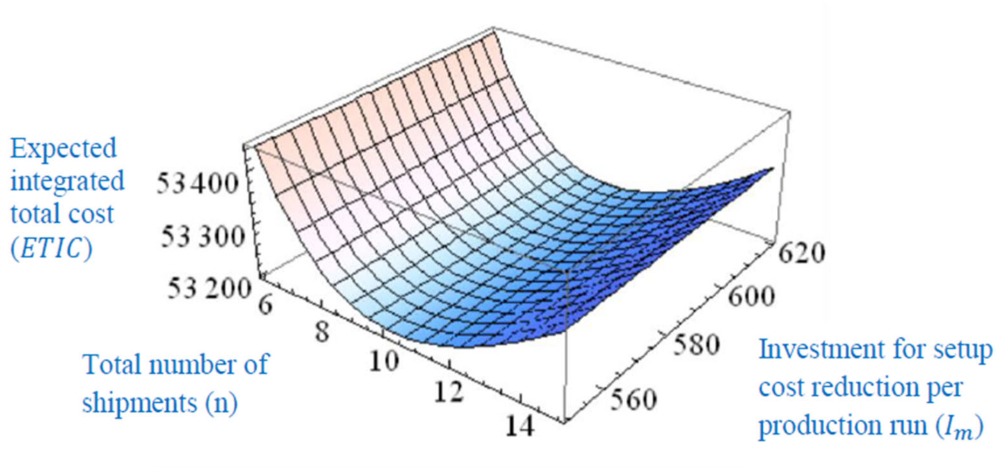

Figure 5. Graphical representation of the expected integrated total cost (*ETIC*) versus total number of shipments (n) and investment for setup cost reduction per production run (I_m) for Example 1, when T is fixed.

Table 2. The optimal solutions for Model –II.

T^*	n^*	u^*	v^*	$q_1{}^*$	$q_2{}^*$	$q_3{}^*$	Q^*	$\widetilde{ETIC}^*$
0.23145	12	3.34	8.66	231	254	208	693	53625.90

number of shipments (n^*) decreases for Example 1 and it remains unaltered for Example 2.

- If the transportation cost of the supplier (F_s) and the manufacturer's warehouse (F_w) increase then the optimal values of all the decision variables i.e.$I_m{}^*$,

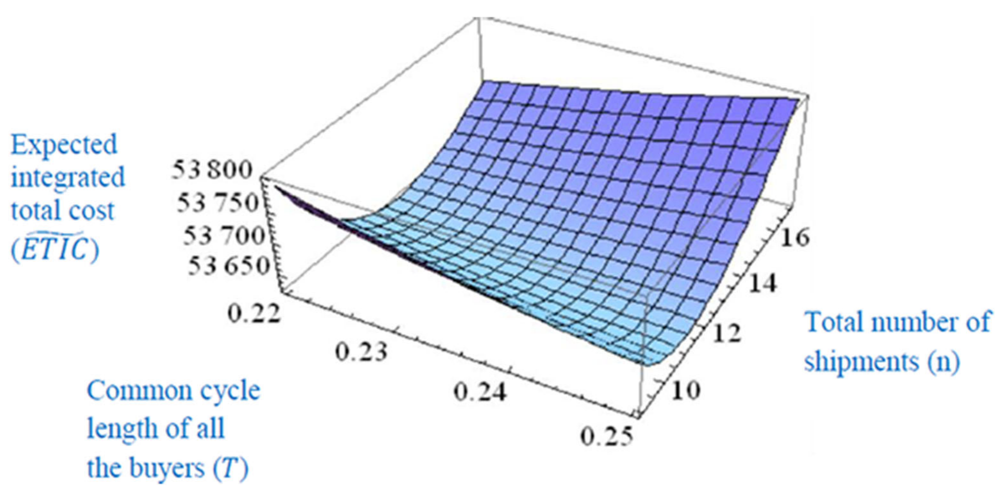

Figure 6. Graphical representation of the expected integrated total cost ($\widetilde{ETIC}$) versus common cycle length of all the buyers (T) and total number of shipments (n) for Example 2.

Table 3. Comparison between Model -I and Model –II.

Optimal results	Model –I (with setup cost reduction)	Model –II (without setup cost reduction)	Save (in %)
n	10	12	–
Setup cost	312.50	2000	84.38
Transportation cost of the producer	360	432	16.67
Expected integrated total cost	53200.50	53625.90	0.8

Table 4. Sensitivity analysis of Examples 1 and 2 when K_m changes.

		Example 1				Example 2		
Parameter	Parameter value	$I_m{}^*$	T^*	n^*	$ETIC^*$	T^*	n^*	$\widetilde{ETIC}^*$
K_m	2500	580.09	0.217491	11	53203.40	0.217491	13	53615.70
	3750	580.09	0.226503	10	53202.10	0.226503	12	53623.10
	5000	**580.09**	**0.23145**	**10**	**53200.50**	**0.23145**	**12**	**53625.90**
	6250	580.09	0.234579	9	53199.30	0.234579	12	53627.20
	7500	580.09	0.236737	9	53198.30	0.236737	12	53628.00

The bold phase indicates the optimal solutions

Table 5. Sensitivity analysis of Examples 1 and 2 when K_r changes.

		Example 1				Example 2		
Parameter	Parameter value	$I_m{}^*$	T^*	n^*	$ETIC^*$	T^*	n^*	$\widetilde{ETIC}^*$
K_r	2000	580.09	0.224384	10	53154.10	0.224384	13	53561.80
	3000	580.09	0.229021	10	53185.40	0.229021	12	53605.00
	4000	**580.09**	**0.23145**	**10**	**53200.50**	**0.23145**	**12**	**53625.90**
	5000	580.09	0.232945	9	53209.40	0.232945	12	53638.30
	6000	580.09	0.233958	9	53215.20	0.233958	12	53646.40

The bold phase indicates the optimal solutions

Table 6. Sensitivity analysis of Examples 1 and 2 when R changes.

		Example 1				Example 2		
Parameter	Parameter value	$I_m{}^*$	T^*	n^*	$ETIC^*$	T^*	n^*	$\widetilde{ETIC}^*$
R	440	580.09	0.232194	8	55202.30	0.232194	10	55685.70
	660	580.09	0.231821	9	54199.80	0.231821	11	54653.70
	880	**580.09**	**0.23145**	**10**	**53200.50**	**0.23145**	**12**	**53625.90**
	1100	580.09	0.231081	10	52205.30	0.231081	13	52603.10
	1320	580.09	0.230713	11	51215.10	0.230713	14	51586.50

The bold phase indicates the optimal solutions

Table 7. Sensitivity analysis of Examples 1 and 2 when A_s changes.

		Example 1				Example 2		
Parameter	Parameter value	$I_m{}^*$	T^*	n^*	$ETIC^*$	T^*	n^*	$\widetilde{ETIC}^*$
A_s	200	580.09	0.23145	9	53112.40	0.23145	12	53555.30
	300	580.09	0.23145	9	53157.00	0.23145	12	53590.90
	400	**580.09**	**0.23145**	**10**	**53200.50**	**0.23145**	**12**	**53625.90**
	500	580.09	0.23145	10	53243.00	0.23145	12	53660.40
	600	580.09	0.23145	10	53284.50	0.23145	12	53694.30

The bold phase indicates the optimal solutions

Table 8. Sensitivity analysis of Examples 1 and 2 when A_w changes.

		Example 1				Example 2		
Parameter	Parameter value	$I_m{}^*$	T^*	n^*	$ETIC^*$	T^*	n^*	$\widetilde{ETIC}^*$
A_w	380	580.09	0.23145	9	53029.00	0.23145	11	53489.70
	570	580.09	0.23145	9	53116.90	0.23145	12	53558.90
	760	**580.09**	**0.23145**	**10**	**53200.50**	**0.23145**	**12**	**53625.90**
	950	580.09	0.23145	10	53280.40	0.23145	12	53690.90
	1140	580.09	0.23145	10	53357.00	0.23145	13	53754.20

The bold phase indicates the optimal solutions

Table 9. Sensitivity analysis of Examples 1 and 2 when A_m^0 changes.

		Example 1				Example 2		
Parameter	Parameter value	$I_m{}^*$	T^*	n^*	$ETIC^*$	T^*	n^*	$\widetilde{ETIC}^*$
A_m^0	1000	363.48	0.23145	9	53104.90	0.23145	10	53246.10
	1500	490.19	0.23145	9	53161.40	0.23145	11	53444.90
	2000	**580.09**	**0.23145**	**10**	**53200.50**	**0.23145**	**12**	**53625.90**
	2500	649.82	0.23145	10	53230.20	0.23145	13	53793.30
	3000	706.80	0.23145	10	53254.10	0.23145	14	53949.60

The bold phase indicates the optimal solutions

T^*, and n^* remain unchanged but $ETIC^*$ and $\widetilde{ETIC}^*$ for Examples 1 and 2 are increased (see tables 13 and 14).

- With an increase in the transportation cost of the producer (F_m), the optimal values T^*, $ETIC^*$, and $\widetilde{ETIC}^*$ for Examples 1 and 2 are increased but n^* decreases and $I_m{}^*$ stays unaltered (see table 15).
- If α increases, T^* remains unchanged but $I_m{}^*$, n^*, and $ETIC^*$ are all decreased (see table 16).

8. Conclusion

In this paper, a manufacturing process is integrated with a remanufacturing process in a multi-layer green supply chain system under setup cost reduction. The orders of the buyers are satisfied with the newly produced and remanufactured items as their qualities are same. If a manufacturing system has a buffer manufacturing system for reworking the defective products, then the main production process may be started after receiving raw materials. In this model, the production system has no buffer manufacturing system. As a result, the remanufacturing/recycling the stock of used items collected during the previous cycle is attempted first, then a fresh production process is taken into consideration. Moreover, lead time between placing and receiving raw materials may occur and then the remanufacturing items meet the demand during lead time. The collection of used returned items as well as the remanufactured and manufactured lots contains a random number of defective items. Numerical results show that the expected integrated total cost is reduced if the producer bears an

Table 10. Sensitivity analysis of Examples 1 and 2 when h_s changes.

		Example 1				Example 2		
Parameter	Parameter value	$I_m{}^*$	T^*	n^*	$ETIC^*$	T^*	n^*	$\widetilde{ETIC}^*$
h_s	0.05	580.09	0.23145	10	53107.20	0.23145	13	53510.50
	0.075	580.09	0.23145	10	53154.50	0.23145	12	53569.00
	0.1	**580.09**	**0.23145**	**10**	**53200.50**	**0.23145**	**12**	**53625.90**
	0.125	580.09	0.23145	9	53245.30	0.23145	12	53681.40
	0.15	580.09	0.23145	9	53289.10	0.23145	11	53735.60

The bold phase indicates the optimal solutions

Table 11. Sensitivity analysis of Examples 1 and 2 when h_w changes.

		Example 1				Example 2		
Parameter	Parameter value	$I_m{}^*$	T^*	n^*	$ETIC^*$	T^*	n^*	$\widetilde{ETIC}^*$
h_w	0.075	580.09	0.23145	12	52896.50	0.23145	14	53249.80
	0.1125	580.09	0.23145	10	53055.50	0.23145	13	53446.60
	0.15	**580.09**	**0.23145**	**10**	**53200.50**	**0.23145**	**12**	**53625.90**
	0.1875	580.09	0.23145	9	53334.60	0.23145	11	53791.80
	0.225	580.09	0.23145	8	53460.00	0.23145	10	53946.90

The bold phase indicates the optimal solutions

Table 12. Sensitivity analysis of Examples 1 and 2 when h_m changes.

		Example 1				Example 2		
Parameter	Parameter value	$I_m{}^*$	T^*	n^*	$ETIC^*$	T^*	n^*	$\widetilde{ETIC}^*$
h_m	0.1	580.09	0.234004	10	53109.30	0.234004	12	53515.40
	0.15	580.09	0.232716	10	53155.40	0.232716	12	53571.30
	0.2	**580.09**	**0.23145**	**10**	**53200.50**	**0.23145**	**12**	**53625.90**
	0.25	580.09	0.230205	9	53244.70	0.230205	12	53679.40
	0.3	580.09	0.228979	9	53288.10	0.228979	12	53731.90

The bold phase indicates the optimal solutions

Table 13. Sensitivity analysis of Examples 1 and 2 when F_s changes.

		Example 1				Example 2		
Parameter	Parameter value	$I_m{}^*$	T^*	n^*	$ETIC^*$	T^*	n^*	$\widetilde{ETIC}^*$
F_s	7.5	580.09	0.23145	10	53197.30	0.23145	12	53623.30
	11.25	580.09	0.23145	10	53198.90	0.23145	12	53624.60
	15	**580.09**	**0.23145**	**10**	**53200.50**	**0.23145**	**12**	**53625.90**
	18.75	580.09	0.23145	10	53202.10	0.23145	12	53627.20
	22.5	580.09	0.23145	10	53203.70	0.23145	12	53628.50

The bold phase indicates the optimal solutions

Table 14. Sensitivity analysis of Examples 1 and 2 when F_w changes.

		Example 1				Example 2		
Parameter	Parameter value	$I_m{}^*$	T^*	n^*	$ETIC^*$	T^*	n^*	$\widetilde{ETIC}^*$
F_w	10	580.09	0.23145	10	53196.20	0.23145	12	53622.40
	15	580.09	0.23145	10	53198.30	0.23145	12	53624.20
	20	**580.09**	**0.23145**	**10**	**53200.50**	**0.23145**	**12**	**53625.90**
	25	580.09	0.23145	10	53202.60	0.23145	12	53627.60
	30	580.09	0.23145	10	53204.80	0.23145	12	53629.40

The bold phase indicates the optimal solutions

Table 15. Sensitivity analysis of Examples 1 and 2 when F_m changes.

		Example 1				Example 2		
Parameter	Parameter value	$I_m{}^*$	T^*	n^*	$ETIC^*$	T^*	n^*	$\widetilde{ETIC}^*$
F_m	18	580.09	0.210680	11	53119.10	0.210680	13	53544.50
	27	580.09	0.221309	10	53160.70	0.221309	13	53586.10
	36	**580.09**	**0.23145**	**10**	**53200.50**	**0.23145**	**12**	**53625.90**
	45	580.09	0.241166	9	53238.60	0.241166	11	53664.00
	54	580.09	0.250505	9	53275.20	0.250505	11	53700.60

The bold phase indicates the optimal solutions

Table 16. Sensitivity analysis of Example 1 when α changes.

		Example 1			
Parameter	Parameter value	$I_m{}^*$	T^*	n^*	$ETIC^*$
α	0.0016	726.97	0.23145	11	53388.10
	0.0024	653.59	0.23145	10	53275.30
	0.0032	**580.09**	**0.23145**	**10**	**53200.50**
	0.0040	519.86	0.23145	9	53147.00
	0.0048	471.20	0.23145	9	53106.50

The bold phase indicates the optimal solutions

additional investment for setup cost reduction. It is observed from sensitivity analyses that the expected integrated total cost will be reduced if the return rate of the used items is increased.

This study may help various manufacturing companies of non-biodegradable products such as computer, printer, mobile, electrical and electronics goods to build their manufacturing policies to reduce their system cost and also the environment waste. This study considers a green supply chain model for a single type of product having constant demand. It can be extended by assuming multiple items with different types of demand patterns.

Appendix A: $J_2 = E\left[(1-\xi)^2\right] = J_1{}^2 + Var(\xi)$

Proof. We have $J_1 = 1 - E[\xi] > 0$. Let $m = E[\xi]$.

Now $E\left[(1-\xi)^2\right] = E(1 - \mathrm{m} - \xi + m)^2$

$$= E[(1-\mathrm{m})^2 - 2(1-\mathrm{m})(\xi - m) + (\xi - m)^2]$$

$$= (1-\mathrm{m})^2 - 2(1-\mathrm{m})(E[\xi] - m) + E[(\xi - m)^2]$$

$$= (1 - E[\xi])^2 + Var(\xi)$$

$$= J_1{}^2 + Var(\xi)$$

Hence the proof. □

Appendix B: *Proof of Theorem* 1

Proof. The Hessian matrix H for the variables $I_m{}^*, T^*,$ and n^* is

$$H = \begin{pmatrix} \frac{\partial^2 ETIC^*}{\partial I_m{}^{*2}} & \frac{\partial^2 ETIC^*}{\partial I_m{}^* \partial T^*} & \frac{\partial^2 ETIC^*}{\partial I_m{}^* \partial n^*} \\ \frac{\partial^2 ETIC^*}{\partial T^* \partial I_m{}^*} & \frac{\partial^2 ETIC^*}{\partial T^{*2}} & \frac{\partial^2 ETIC^*}{\partial T^* \partial n^*} \\ \frac{\partial^2 ETIC^*}{\partial n^* \partial I_m{}^*} & \frac{\partial^2 ETIC^*}{\partial n^* \partial T^*} & \frac{\partial^2 ETIC^*}{\partial n^{*2}} \end{pmatrix}$$

where $ETIC^* = ETIC(I_m{}^*, T^*, n^*)$.

The partial derivatives of $ETIC^*$ with respect to $I_m{}^*, T^*,$ and n^* are as follows.

$$\frac{\partial ETIC^*}{\partial I_m{}^*} = \frac{1}{n^* T^*}\left(-\alpha A_m^0 e^{-\alpha I_m{}^*} + 1\right)$$

$$\frac{\partial ETIC^*}{\partial T^*} = -\frac{1}{n^* T^{*2}}\left[J_0 + A_m^0 e^{-\alpha I_m{}^*} + I_m{}^* + n^*\left(F_m + \sum_{i=1}^{\lambda} A_{bi}\right)\right] + \frac{1}{2}\left[n^*(h_s J_3 + h_w J_4 + h_m J_5) + h_m J_6 + \sum_{i=1}^{\lambda} h_{bi} d_i\right]$$

$$\frac{\partial ETIC^*}{\partial n^*} = -\frac{1}{n^{*2} T^*}\left(J_0 + A_m^0 e^{-\alpha I_m{}^*} + I_m{}^*\right) + \frac{T^*}{2}(h_s J_3 + h_w J_4 + h_m J_5)$$

At $I_m{}^*, T^*, n^*, \frac{\partial ETIC^*}{\partial I_m{}^*} = 0, \frac{\partial ETIC^*}{\partial T^*} = 0$ and $\frac{\partial ETIC^*}{\partial n^*} = 0$ which gives respectively

$$1 - \alpha A_m^0 e^{-\alpha I_m{}^*} = 0 \tag{12}$$

$$\frac{1}{n^* T^{*2}}\left[J_0 + A_m^0 e^{-\alpha I_m{}^*} + I_m{}^* + n^*\left(F_m + \sum_{i=1}^{\lambda} A_{bi}\right)\right] = \frac{1}{2}\left[n^*(h_s J_3 + h_w J_4 + h_m J_5) + h_m J_6 + \sum_{i=1}^{\lambda} h_{bi} d_i\right] \tag{13}$$

$$\frac{1}{n^{*2}T^*}\left(J_0 + A_m^0 e^{-\alpha I_m{}^*} + I_m{}^*\right) = \frac{T^*}{2}\left(h_s J_3 + h_w J_4 + h_m J_5\right) \tag{14}$$

The second order partial derivatives are

$$\frac{\partial^2 ETIC^*}{\partial I_m{}^{*2}} = \frac{\alpha^2 A_m^0 e^{-\alpha I_m{}^*}}{n^* T^*} > 0$$

$$\frac{\partial^2 ETIC^*}{\partial T^{*2}} = \frac{1}{T^*}\left[n^*(h_s J_3 + h_w J_4 + h_m J_5) + h_m J_6 + \sum_{i=1}^{\lambda} h_{bi} d_i\right] > 0, \text{[using equation (13)]}$$

$$\frac{\partial^2 ETIC^*}{\partial n^{*2}} = \frac{T^*}{n^*}(h_s J_3 + h_w J_4 + h_m J_5) > 0, \text{[using equation (14)]}$$

$$\frac{\partial^2 ETIC^*}{\partial I_m{}^* \partial T^*} = \frac{\partial^2 ETIC^*}{\partial T^* \partial I_m{}^*} = 0, \text{[using equation (12)]}$$

$$\frac{\partial^2 ETIC^*}{\partial I_m{}^* \partial n^*} = \frac{\partial^2 ETIC^*}{\partial n^* \partial I_m{}^*} = 0, \text{[using equation (12)]}$$

$$\frac{\partial^2 ETIC^*}{\partial T^* \partial n^*} = \frac{\partial^2 ETIC^*}{\partial n^* \partial T^*} = h_s J_3 + h_w J_4 + h_m J_5, \text{[using equation (14)]}$$

The first, second and third order principal minors of $|H|$ at $(I_m{}^*, T^*, n^*)$ are respectively as follows.

$$\det(H_{11}) = \det\left(\frac{\partial^2 ETIC^*}{\partial I_m{}^{*2}}\right) = \frac{\alpha^2 A_m^0 e^{-\alpha I_m{}^*}}{n^* T^*} > 0$$

$$\det(H_{22}) = \det\begin{pmatrix} \frac{\partial^2 ETIC^*}{\partial I_m{}^{*2}} & 0 \\ 0 & \frac{\partial^2 ETIC^*}{\partial T^{*2}} \end{pmatrix}$$

$$= \frac{\alpha^2 A_m^0 e^{-\alpha I_m{}^*}}{n^* T^{*2}}\left[n^*(h_s J_3 + h_w J_4 + h_m J_5) + h_m J_6 + \sum_{i=1}^{\lambda} h_{bi} d_i\right] > 0$$

$$\det(H_{33}) = \det\begin{pmatrix} \frac{\partial^2 ETIC^*}{\partial I_m{}^{*2}} & 0 & 0 \\ 0 & \frac{\partial^2 ETIC^*}{\partial T^{*2}} & \frac{\partial^2 ETIC^*}{\partial T^* \partial n^*} \\ 0 & \frac{\partial^2 ETIC^*}{\partial n^* \partial T^*} & \frac{\partial^2 ETIC^*}{\partial n^{*2}} \end{pmatrix}$$

$$= \frac{\partial^2 ETIC^*}{\partial I_m{}^{*2}}\left\{\frac{\partial^2 ETIC^*}{\partial T^{*2}} \cdot \frac{\partial^2 ETIC^*}{\partial n^{*2}} - \left(\frac{\partial^2 ETIC^*}{\partial T^* \partial n^*}\right)^2\right\}$$

$$= \frac{\alpha^2 A_m^0 e^{-\alpha I_m{}^*}}{n^{*2} T^*}(h_s J_3 + h_w J_4 + h_m J_5)\left(h_m J_6 + \sum_{i=1}^{\lambda} h_{bi} d_i\right) > 0.$$

Thus, all the principal minors of $|H|$ at $(I_m{}^*, T^*, n^*)$ are positive. Therefore, the Hessian matrix H for the variables $I_m{}^*,\ T^*,$ and n^* is positive definite. Thus, the expected integrated total cost for Model-I has a global minimum at the optimal solution $(I_m{}^*, T^*, n^*)$.

Hence the proof. □

Appendix C: *Proof of Theorem* 2

Proof. The Hessian matrix $\tilde{H}$ for the variables T^* and n^* is

$$\tilde{H} = \begin{pmatrix} \frac{\partial^2 \widetilde{ETIC}^*}{\partial T^{*2}} & \frac{\partial^2 \widetilde{ETIC}^*}{\partial T^* \partial n^*} \\ \frac{\partial^2 \widetilde{ETIC}^*}{\partial n^* \partial T^*} & \frac{\partial^2 \widetilde{ETIC}^*}{\partial n^{*2}} \end{pmatrix}$$

where $\widetilde{ETIC}^* = \widetilde{ETIC}(T^*, n^*)$.

The partial derivatives of $\widetilde{ETIC}^*$ with respect to T^* and n^* are as follows.

$$\frac{\partial \widetilde{ETIC}^*}{\partial T^*} = -\frac{1}{n^* T^{*2}}\left[J_0 + A_m^0 + n^*\left(F_m + \sum_{i=1}^{\lambda} A_{bi}\right)\right] + \frac{1}{2}\left[n^*(h_s J_3 + h_w J_4 + h_m J_5) + h_m J_6 + \sum_{i=1}^{\lambda} h_{bi} d_i\right]$$

$$\frac{\partial \widetilde{ETIC}^*}{\partial n^*} = -\frac{1}{n^{*2} T^*}\left(J_0 + A_m^0\right) + \frac{T^*}{2}(h_s J_3 + h_w J_4 + h_m J_5)$$

At T^*, n^*, $\frac{\partial \widetilde{ETIC}^*}{\partial T^*} = 0$ and $\frac{\partial \widetilde{ETIC}^*}{\partial n^*} = 0$ which gives respectively

$$\frac{1}{n^* T^{*2}}\left[J_0 + A_m^0 + n^*\left(F_m + \sum_{i=1}^{\lambda} A_{bi}\right)\right] = \frac{1}{2}\left[n^*(h_s J_3 + h_w J_4 + h_m J_5) + h_m J_6 + \sum_{i=1}^{\lambda} h_{bi} d_i\right] \tag{15}$$

$$\frac{1}{n^{*2} T^*}\left(J_0 + A_m^0\right) = \frac{T^*}{2}(h_s J_3 + h_w J_4 + h_m J_5) \tag{16}$$

The second order partial derivatives are

$$\frac{\partial^2 \widetilde{ETIC}^*}{\partial T^{*2}} = \frac{1}{T^*}\left[n^*(h_s J_3 + h_w J_4 + h_m J_5) + h_m J_6 + \sum_{i=1}^{\lambda} h_{bi} d_i\right] > 0, \text{[using equation (15)]}$$

$$\frac{\partial^2 \widetilde{ETIC}^*}{\partial n^{*2}} = \frac{T^*}{n^*}(h_s J_3 + h_w J_4 + h_m J_5) > 0, \text{[using equation (16)]}$$

$$\frac{\partial^2 \widetilde{ETIC}^*}{\partial T^* \partial n^*} = \frac{\partial^2 \widetilde{ETIC}^*}{\partial n^* \partial T^*} = h_s J_3 + h_w J_4 + h_m J_5, \text{[using equation (16)]}$$

$$\text{Now, } \det(\tilde{H}) = \frac{\partial^2 \widetilde{ETIC}^*}{\partial T^{*2}} \cdot \frac{\partial^2 \widetilde{ETIC}^*}{\partial n^{*2}} - \left(\frac{\partial^2 \widetilde{ETIC}^*}{\partial T^* \partial n^*}\right)^2$$

$$= \frac{(h_s J_3 + h_w J_4 + h_m J_5)}{n^*}\left(h_m J_6 + \sum_{i=1}^{\lambda} h_{bi} d_i\right) > 0.$$

Therefore, the Hessian matrix $\tilde{H}$ is positive definite. Thus, the expected integrated total cost for Model-II has a global minimum at the optimal solution (T^*, n^*).

Hence the proof. □

List of symbols

Parameters

λ	Total number of buyers
d_i	Demand rate (units/time) of the i th buyer, $i = 1, 2, \ldots, \lambda$
D	Average demand (units/time), where $D = \sum_{i=1}^{\lambda} d_i$
R	Returned rate (units/time) of the used items
K_m	Regular production rate (units/time) of the producer
K_r	Remanufacturing rate (units/time) of the producer
ξ	Random variable that indicates the percentage of defective items in the collection of used returned items
β	Random variable that indicates the percentage of defectives produced during the remanufacturing process
γ	Random variable that indicates the percentage of defectives produced during the regular production process
$f(\xi)$	Probability density function of ξ
$g_1(\beta)$	Probability density function of β
$g_2(\gamma)$	Probability density function of γ
$E[.]$	Mathematical expectation
A_s	Ordering cost (\$/order) of the supplier
A_w	Ordering cost (\$/order) of the manufacturer's warehouse
A_m^0	Initial setup cost (\$/setup) of the producer
A_{bi}	Ordering cost (\$/order) of the i th buyer, where $i = 1, 2, \ldots, \lambda$
p_s	Purchasing cost (\$/unit) of the supplier
p_w	Purchasing cost (\$/unit) of the manufacturer's warehouse
p_b	Purchasing cost (\$/unit) of each buyer
C_a	Accusation cost (\$/unit) (cost of used returned items) of the manufacturer's warehouse
C_u	Production cost (\$/unit) of the producer
C_v	Remanufacturing cost (\$/unit) of the producer
h_s	Holding cost (\$/unit/time) of the supplier
h_w	Holding cost (\$/unit/time) of the manufacturer's warehouse
h_m	Holding cost (\$/unit/time) of the producer
h_{bi}	Holding cost (\$/unit/time) of the ith buyer, $i = 1, 2, \ldots, \lambda$
S_r	Screening rate (units/unit time) of the manufacturer's warehouse
S_c	Screening cost (\$/unit) of the manufacturer's warehouse
C_d	Dispose cost (\$/unit) of the manufacturer's warehouse
C_r	Rework cost (\$/unit) of the producer
F_s	Fixed transportation cost (\$/order) of the supplier
F_w	Fixed transportation cost (\$/order) of the manufacturer's warehouse
F_m	Transportation cost (\$/shipment) of the producer
I_s	Idle cost (\$/unit time) of the supplier

Decision variables

u	Fraction of total number of shipments of the remanufacturing products
v	Fraction of total number of shipments of the newly manufactured products
n	Total number of shipments i.e., $n = u + v$
T	Common cycle length of all the buyers.
q_i	Order lot size (units) of the i th buyer, where $q_i = d_i T$, $i = 1, 2, \ldots, \lambda$
Q	Delivery lot size (units/shipment) of the producer, where $Q = \sum_{i=1}^{\lambda} q_i$
I_m	Investment for setup cost reduction (\$/production run) of the producer
$ETIC$	The expected integrated total cost (\$/unit time) for Model-I
$\widetilde{ETIC}$	The expected integrated total cost (\$/unit time) for Model-II

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