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Dying AdS Schwarzschild black hole

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The criteria for thermal stability of a most general quantum black hole derived by us appeared in the form of a series of inequalities connecting second-order derivatives of black hole mass with respect to its parameters, which determine the mass of the black hole. These nullify the concept of positivity of specific heat as the sole criteria for thermal stability. Using this most general stability criterion, we prove here that AdS Schwarzschild black holes are no longer stable anywhere in their parameter space if cosmological constant is allowed to vary. We also calculate the fluctuations of both horizon area and cosmological constant of this black hole. We calculate specific heat of it and compare this with Hawking's prediction.

Keywords: Quasi stable black hole; non Hawking phase transition; quantum black hole.

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1. Introduction

Hawking through his semi-classical theory showed that black holes had radiated.[1] Thus, a black hole accretes and radiates simultaneously. He showed that asymptotically flat Schwarzschild black hole (AFSBH) was thermally unstable due to negative specific heat.[2] In fact, the sign of specific heat was the only determining criterion for thermal stability in his theory. He later showed that AdS Schwarzschild black hole (AdSSBH) would be stable only in certain regime of parameter space.[3,4] He also established the possible phase transition for this black hole, marked by the change in the sign of specific heat.[4] Hawking there assumed the cosmological constant to be fixed.

Black holes were still classical in Hawking's semi-classical theory. We addressed this issue earlier[5,6] and treated black holes as quantum entity. We derived the stability criteria for a most general quantum black hole with any number of parameters in arbitrary dimensional spacetime.[6] They appeared in the form of a series of inequalities connecting second-order derivatives of black hole mass with respect to its parameters. Some black holes never satisfy all the stability criteria together, but satisfy some of the stability criteria in certain region of parameter space. These black holes are known as "quasi-stable black holes" in that regime. We showed[7–9] that these black holes have bounded fluctuations for some of their parameters in certain region of parameter space. In fact, it was also[8] found that these black holes had tendency to reduce the decay rate under hawking radiation.

We showed[8,9] that the asymptotically flat black holes, even with electric charge, angular momentum, are not stable anywhere in parameter space. They are actually quasi-stable. But their respective AdS versions are stable in certain regime of parameter space.[7] Those AdS spaces are of fixed cosmological constant. But cosmological constant evidently varies.[10,11] Thus, it is necessary to revisit the thermodynamic stability of the AdS black holes in this context of varying cosmological constant. We do this here in case of AdS Schwarzschild black hole (AdSSBH), resulting a drastic change in its thermodynamic behavior. We show in this paper that AdSSBH cannot be stable, actually would be quasi-stable, if cosmological constant is allowed to fluctuate. We also discuss the other features of AdSSBH as a quasi-stable black hole. In fact, we also calculate specific heat of this black hole and compare it with Hawking's result. We will see here that application of our stability criteria in context of the simplest possible AdS black hole, i.e. AdSSBH gives an entirely unknown, unexploited result, i.e. the thermal (quasi) stability of AdSSBH.

This paper is organized as follows. In Sec. 2, we recapitulate some of our earlier works briefly for sake of completeness, with emphasis on cosmological constant fluctuating AdSSBH. We also show there that this black hole is actually quasi-stable under Hawking radiation. In Sec. 3, we calculate the fluctuations of both the parameters of AdSSBH. We also discuss on possible phase transition. In Sec. 4, we calculate specific heat and compare our obtained results with Hawking's semi-classical theory. In Sec. 5, we summarize our work with possible outlooks.

2. Quasi-Stability of AdS Schwarzschild Black Hole

We know so far that AdSSBH is thermally stable in certain region of parameter space. Thus once it is in that region, it can prevent itself from decaying away under Hawking radiation. But cosmological constant is assumed there to be fixed. In this paper, we allow it to fluctuate. Once we do that, we have to consider this fluctuation to construct the grand canonical partition function (Z_G) for AdSSBH. We already[5,6] knew how to construct Z_G of generic quantum black hole. We only assumed there that the mass of a black hole was function of its parameters, which were fluctuating. Similarly, we treat cosmological constant as a parameter of AdSSBH as mass of it is a function of fluctuating cosmological constant. Thus, mass of AdSSBH (M) is given as, $M = M(A, \tilde{\Lambda})$. Here, A denotes the horizon area of the black hole. Cosmological constant is negative for AdS space. We choose $\tilde{\Lambda}$ to be negative of that and hence $\tilde{\Lambda}$ is positive. Now, it is to be noted that the mass of an AdS black hole is in general believed to be the enthalpy of the spacetime.[12] Thus, it is naively expected that term like PV should be added with the mass term $(M(A, \tilde{\Lambda}))$ to get the correct expression, where P is thermodynamic pressure of the black hole, identified to be equivalent with the cosmological constant[13,14] and V is the associated volume. Of course this identification is required for extended phase–space analysis for thermodynamics of AdS black holes.[15] But this is not at all required for our analysis and we in fact do not bother about such identification at all. The associated volume (V) is the volume excluded by the black hole horizon from a spatial slice exterior to the black hole.[16,17] Hence, this volume term does not bother us at all as we calculate everything on the horizon of the black hole.[5] We will see latter that all the second-order derivatives of black hole mass and in fact mass of the black hole, its area, etc. are all evaluated on the horizon. We study the local thermodynamics of the black holes, around its horizon. Hence, we do not have to bother about the enthalpy of the spacetime at all and our mass term $(M(A, \tilde{\Lambda}))$ is appropriate enough to describe what we are going to do in correct manner.

In any full-fledged theory of quantum gravity, A and $\tilde{\Lambda}$ are some discrete quantities.[18] Thus they are collections of their respective quanta. Now we can consider an AdSSBH to be immersed in a heat bath, with which it can exchange mass and $\tilde{\Lambda}$. Black holes exchange their parameters with heat bath, i.e. its surrounding in form of respective quanta. Thus, we can write down the grand canonical partition function (Z_G) as summation over possible eigenstates with appropriate weightage.[19] We can convert this summation, with the help of Poisson's resummation formula,[20] into integration and determine the criteria for thermal stability. AdSSBH, in thermal equilibrium, is represented by the saddle point $(\bar{A}, \bar{\tilde{\Lambda}})$. $\bar{A}$ denotes horizon area (A) at equilibrium and so on. It is shown earlier[5] that this partition function turned out to be integration over the space of fluctuations $a = (A - \bar{A})$, $\lambda = (\tilde{\Lambda} - \bar{\tilde{\Lambda}})$ around the saddle point and is given as[5]

$$Z_G \approx \int da\, d\lambda \exp\left(-\frac{\beta}{2}\left[\left(M_{AA} - \frac{S_{AA}}{\beta}\right)a^2 + (M_{\tilde{\Lambda}\tilde{\Lambda}})\lambda^2 + (2M_{A\tilde{\Lambda}})a\lambda\right]\right). \quad (1)$$

Here, $M_{AA} \equiv \frac{\partial^2 M}{\partial A^2}$, $M_{A\tilde{\Lambda}} \equiv \frac{\partial^2 M}{\partial A \partial \tilde{\Lambda}}$, etc. and they are evaluated at the saddle point. $S(A)$ is the micro-canonical entropy of the black hole and is equal to one quarter of its horizon area in units of Planck area. We take four times Planck area equals to unity for the sake of simplicity and can be brought back by dimensional analysis whenever required.

The convexity of the above integral leads to the criteria for thermal stability of the black hole[5] and is given as

$$(\beta M_{AA} - S_{AA}) > 0, \quad M_{\tilde{\Lambda}\tilde{\Lambda}} > 0, \quad \left(M_{\tilde{\Lambda}\tilde{\Lambda}}(\beta M_{AA} - S_{AA}) - \beta(M_{A\tilde{\Lambda}})^2\right) > 0.$$

We have realistically assumed that (inverse) temperature β is $\left(\equiv \frac{S_A}{M_A}\right)$, which is positive.

The mass (M) of AdSSBH depends on its parameters as[4]

$$M = \frac{\sqrt{A}}{4\sqrt{\pi}} + \frac{\tilde{\Lambda} A^{3/2}}{48\pi^{3/2}}. \tag{2}$$

We can now easily calculate the value of M_A, equals to temperature T, from the above relationship and it turns out to be

$$\begin{aligned} M_A &= \frac{1}{8\sqrt{\pi}\sqrt{A}} + \frac{\tilde{\Lambda} A^{1/2}}{32\pi^{3/2}} \\ &= T. \end{aligned} \tag{3}$$

Thus we see that temperature of AdSSBH is positive irrespective of its position in parameter space. Hence the positivity of temperature cannot restrict the parameter space of a real AdSSBH.

We can now calculate various second-order derivatives of the black hole mass (M) with respect to its parameters from Eq. (2). On calculation, this turns out that

$$M_{\tilde{\Lambda}\tilde{\Lambda}} = 0, \quad M_{A\tilde{\Lambda}} = \frac{A^{1/2}}{32\pi^{3/2}}, \quad M_{AA} = -\frac{1}{16\sqrt{\pi}A^{3/2}} + \frac{\tilde{\Lambda}}{64\pi^{3/2}A^{1/2}},$$

$$\therefore \ \left(M_{\tilde{\Lambda}\tilde{\Lambda}}(\beta M_{AA} - S_{AA}) - \beta(M_{A\tilde{\Lambda}})^2\right) = -\beta(M_{A\tilde{\Lambda}})^2 < 0.$$

Thus, M_{AA} is positive if $A > \frac{4\pi}{\tilde{\Lambda}}$. But $M_{\tilde{\Lambda}\tilde{\Lambda}}$ and $\left(M_{\tilde{\Lambda}\tilde{\Lambda}}(\beta M_{AA} - S_{AA}) - \beta(M_{A\tilde{\Lambda}})^2\right)$ are never positive. Hence this black hole can never be stable under Hawking radiation. It is only quasi-stable in the region $A > \frac{4\pi}{\tilde{\Lambda}}$.

In fact this is the region of thermal stability[4] if $\tilde{\Lambda}$ is not allowed to fluctuate. Thus, we see that fluctuation in $\tilde{\Lambda}$ makes AdSSBH into a decaying black hole under Hawking radiation.

3. Thermal Fluctuations and Quasi-Stable Phase Transition of AdSSBH

We have already seen that[8,9] quasi-stable black holes possess bounded fluctuations for some of their parameters in certain regions of parameter spaces. So, same is expected to be true here for AdSSBH with varying cosmological constant. It is also known to us[8] how fluctuations are related to stability criteria. In fact, we also know[8] how to calculate fluctuations in case of quasi-stable black holes. $\Delta(\tilde{\Lambda})^2$ measures the fluctuation of cosmological constant from its equilibrium value. It is mathematically expressed as,[7,8] $\Delta(\tilde{\Lambda})^2 = \frac{\int da\, d\lambda\, \lambda^2 f(a,\lambda)}{\int da\, d\lambda\, f(a,\lambda)}$; where $f(a,\lambda) = \exp\left(-\frac{\beta}{2}\left[\left(M_{AA} - \frac{S_{AA}}{\beta}\right)a^2 + (2M_{A\tilde{\Lambda}})a\lambda\right]\right)$. Similarly, $\Delta(A)^2$ is defined. Now, $\left(M_{\tilde{\Lambda}\tilde{\Lambda}}(\beta M_{AA} - S_{AA}) - \beta(M_{A\tilde{\Lambda}})^2\right)$ is always negative. Hence we can conclude that

(1) $\Delta(A)^2$ always blows up as $M_{\tilde{\Lambda}\tilde{\Lambda}}$ is identically zero.
(2) $\Delta(\tilde{\Lambda})^2$ is bounded if $(\beta M_{AA} - S_{AA})$ is negative, i.e. $A < \frac{4\pi}{\tilde{\Lambda}}$, otherwise it blows up.

AdSSBH, having variable cosmological constant, ultimately decays under Hawking radiation. Fluctuation in area is always unbounded for this black hole. It gradually becomes smaller and smaller in size. Suppose the size of the black hole is initially such that, $A\tilde{\Lambda}$ is greater than 4π. But due to Hawking radiation, area reduces by a huge amount such that $A\tilde{\Lambda}$ becomes lesser than 4π. But in this regime of parameter space, cosmological constant does not fluctuate much and hence $A\tilde{\Lambda}$ remains lesser than 4π. Thus, it decays away and heads towards a black hole with certain minimum area.[21]

It is unexpected to think that both the fluctuations occur simultaneously in same pace. In fact, we can expect things to happen in other way, i.e. one fluctuation dominates over the other fluctuation region wise in parameter space. In fact, our previous analysis supports this view. Suppose fluctuation in cosmological constant is extremely tiny and hence dominating fluctuation here is the fluctuation in horizon area. In this situation, grand canonical partition function (1) is reduced as

$$Z_G \approx \int da \exp\left(-\frac{\beta}{2}\left[\left(M_{AA} - \frac{S_{AA}}{\beta}\right)a^2\right]\right). \tag{4}$$

Thus this partition function is converging only if $\left(-\frac{\beta}{2}\left[\left(M_{AA} - \frac{S_{AA}}{\beta}\right)\right]\right)$ is positive, i.e. $A > \frac{4\pi}{\tilde{\Lambda}}$. In fact, this regime of parameter space is also the regime where fluctuation in horizon area is bounded. It is to note that this is exactly the well-known case with AdSSBH with fixed cosmological constant.[4]

On the other hand, if fluctuation in horizon area is extremely tiny, then fluctuation in cosmological constant becomes the dominating one. In this case, grand canonical partition function (1) is reduced as

$$Z_G \approx \int d\lambda \exp\left(-(M_{\tilde{\Lambda}\tilde{\Lambda}})\lambda^2\right). \tag{5}$$

The above partition function blows up as $M_{\tilde{\Lambda}\tilde{\Lambda}}$ is identically zero. Thus, fluctuation in cosmological constant blows up even more sharply.

It is interesting to note as conclusion that any nonzero, even bounded, fluctuation in cosmological constant triggers an unbounded fluctuation in horizon area. This in fact causes the ultimate decay of this black hole. On the other hand, non-vanishing fluctuation in horizon area helps cosmological constant to have a decent bounded fluctuation in the regime $A < \frac{4\pi}{\tilde{\Lambda}}$. Thus these two fluctuations are highly correlated in thermodynamic sense.

We know that quasi-stable black holes have many interesting new types of phases. We have already worked out in detail about those phases.[22,23] In brief, each of the fluctuations of the parameters can at most hold two distinguished phases that are connected through a phase transition. This phase transition is marked by the change in associated fluctuation from its bounded nature to its divergence. $\Delta(A)^2$ always blows up and hence it does not connect two different phases and hence no associated phase transition occurs. But $\Delta(\tilde{\Lambda})^2$ changes its bounded nature and becomes unbounded as it crosses $A\tilde{\Lambda} = 4\pi$ curve. Thus, it indicates a phase transition.

We can define $\bar{\alpha} \equiv \beta\alpha$, where $\alpha \equiv \frac{\partial M}{\partial \tilde{\Lambda}}$ and is evaluated on the horizon. This quantity α is analogous to electric potential in case of charged black hole, i.e. if we replace $\tilde{\Lambda}$ by charge(Q). This $\bar{\alpha}$ determines the equilibrium between black hole and rest of the universe in cosmological sector, i.e. their sharing of quanta corresponding to cosmological constant. We can again define average cosmological constant ($\bar{\tilde{\Lambda}}$) as, $\bar{\tilde{\Lambda}} \equiv \frac{\partial(\ln(Z_G))}{\partial \bar{\alpha}}$ and is again evaluated on the horizon. Thus, physical quantity cosmological constant density, analogous to electric capacitance in case of charge, is denoted and defined as, $S_\alpha \equiv \beta \cdot \frac{\partial \bar{\tilde{\Lambda}}}{\partial \bar{\alpha}}$. This equals to $\beta \cdot \Delta(\tilde{\Lambda})^2$, if $\Delta(\tilde{\Lambda})^2$ converges. S_α becomes zero at the point of phase transition, i.e. when M_{AA} vanishes. Sign change in S_α denotes the phase transition. Thus, AdSSBH undergoes quasi-phase transition if cosmological constant is allowed to fluctuate.

The decay of any black hole is approximately governed by Stefan–Boltzmann law as the profile of black hole radiation is approximately same as that of a black body. So, luminosity (L), the power radiated per unit surface area, varies with its temperature (T) as $L \propto T^4$. Temperature is here a function of cosmological constant and area and any fluctuation in them would make temperature fluctuating as well. This in turn makes luminosity fluctuating, too. Thus, the fluctuation in luminosity is given as

$$\Delta L \propto \left(\frac{\partial T}{\partial A}\Delta A + \frac{\partial T}{\partial \tilde{\Lambda}}\Delta\tilde{\Lambda}\right).$$

Now, $T \propto M_A$ and hence

$$\frac{\partial T}{\partial A} \propto M_{AA}, \qquad \frac{\partial T}{\partial \tilde{\Lambda}} \propto M_{A\tilde{\Lambda}}.$$

$$\therefore\ \Delta L \propto \left(M_{AA}\Delta A + M_{A\tilde{\Lambda}}\Delta\tilde{\Lambda}\right).$$

Thus, we see the expression within the above parenthesis determines the sign of ΔL, i.e. whether decay rate would increase or not. Now ΔA is always large negative and $M_{A\tilde{\Lambda}}$ is always positive. In the regime $A\tilde{\Lambda} > 4\pi$, M_{AA} is positive and $\Delta\tilde{\Lambda}$ is large. Hence the term $M_{AA}\Delta A$ is negative in this particular regime and consequently tries to slow down the decay process. In fact in this regime $\tilde{\Lambda}$ is also likely to be reduced due to bubble emission[24] and hence $\Delta\tilde{\Lambda}$ is negative. Thus the sign of the term $M_{A\tilde{\Lambda}}\Delta\tilde{\Lambda}$ is negative and consequently it reduces the decay rate. A keeps on decreasing along with $\tilde{\Lambda}$. Hence the black hole once crosses the curve $A\tilde{\Lambda} = 4\pi$, i.e. phase transition occurs. After that in the regime $A\tilde{\Lambda} < 4\pi$, $\tilde{\Lambda}$ does not fluctuate much but area keeps on decreasing, maintaining the inequality $A\tilde{\Lambda} < 4\pi$. This guarantees that reentrant phase transition[25] does not occur here, unlike in case of asymptotically flat rotating charged quasi-stable black holes.[23] Thus the black hole tries to end up with having a minimum area,[21] but having substantial amount of cosmological constant. In fact, in the regime $A\tilde{\Lambda} < 4\pi$, M_{AA} is negative where $\Delta\tilde{\Lambda}$ is very small. Thus, ΔL is positive in this regime and it makes sure that black hole would eventually die out.

4. Comparison with Hawking's Semi-Classical Theory

Hawking first discovered black hole radiation.[1,2] He also predicted the thermal instability of asymptotically flat Schwarzschild black hole (AFSBH) due to its negative specific heat.[4] He showed that specific heat blew up at phase transition. In fact, in his case-by-case study approach, he used the sign of specific heat as the only criterion to determine the thermal stability.[4] We show here explicitly that this is not the ultimate story at all.

Now, specific heat (C) is defined as, $C \equiv \frac{\partial M}{\partial T}$. Both M and T are here functions of $\tilde{\Lambda}$ and A. Thus we can write $C = \frac{\partial M}{\partial A} \cdot \frac{\partial A}{\partial T} + \frac{\partial M}{\partial \tilde{\Lambda}} \cdot \frac{\partial \tilde{\Lambda}}{\partial T}$.

We can easily calculate the values of $\frac{\partial A}{\partial T}$ and $\frac{\partial \tilde{\Lambda}}{\partial T}$ from Eq. (3) and they are given, respectively, as

$$\frac{\partial A}{\partial T} = 1 \Big/ \left(-\frac{1}{16\sqrt{\pi} A^{3/2}} + \frac{\tilde{\Lambda}}{64\pi^{3/2} A^{1/2}} \right) \qquad \text{and} \qquad \frac{\partial \tilde{\Lambda}}{\partial T} = \frac{32\pi^{3/2}}{A^{1/2}}.$$

On calculation, we find that

$$C = 4A\left(\frac{A\tilde{\Lambda}}{2\pi} + 1\right) \Big/ 3\left(\frac{A\tilde{\Lambda}}{4\pi} - 1\right).$$

Following the above expression, we get

(1) $C < 0$ if $A\tilde{\Lambda} < 4\pi$,
(2) $C > 0$ if $A\tilde{\Lambda} > 4\pi$.

So, according to Hawking's prediction, in the region $A\tilde{\Lambda} > 4\pi$, AdSSBH is thermally stable as specific heat is positive. But we have already shown explicitly that this black hole, having varying cosmological constant, cannot be stable anywhere in its accessible parameter space. Thus, we see explicitly that positivity of specific heat cannot be the only criteria for thermal stability of black holes.

In his semi-classical theory, Hawking treated only matters as quantum entity, but black hole was still classical. But we treated both of them as quantum entity. Thus, we can expect even theoretically that our result should mismatch with Hawking's prediction at some point of time. In this paper, we show this explicitly in case of AdSSBH. It is true for any black hole having parameter in addition to its horizon area. In fact we treat cosmological constant in this paper as a parameter of the black hole. Hence, Hawking's prediction for region of stability for AdSSBH is actually region of quasi-stability if cosmological constant is allowed to vary. Hawking's prediction matches with our results only in case of black holes with horizon area as only parameter, e.g. AFSBH or AdSSBH with fixed cosmological constant.[4] Actually, predictions regarding thermal stability of black holes with multiple parameters, be it stable or quasi-stable, are not correct if one tries to apply Hawking's semi-classical theory.

5. Discussion

We show here that AdSSBH with variable cosmological constant ultimately decays under Hawking radiation. The region of quasi-stability of this black hole would exactly be the region of stability if cosmological constant remains fixed. This is in fact expected too as otherwise our result neither can be trusted and at the same time loses its platform to be verified in appropriate known limit. It, like all other quasi-stable black holes, tries to resist the decay process of Hawking radiation. Our analysis holds for macroscopic black holes. But close to end state of a black hole, we have to solve the complicated Hamiltonian constraint. But it is true that there would be some remnant of a black hole at end state, with a minimum area according to theory like LQG. This fact has been mentioned in Ref. 21 in which it is concluded that these remnants could form component of dark matter as well. In this sense, our analysis may have some impacts on dark matter physics.

Now, the AdS/CFT correspondence tells that an asymptotically AdS black hole is dual to a strongly coupled gauge theory at finite temperature.[26–29] It is possible to analyze the strongly correlated condensed matter physics using AdS/CFT correspondence. Thus our calculations for this quasi-stable black hole may have some imprints to condensed matter physics as well, especially as we are allowing cosmological constant to vary. This may certainly give new interesting results.

In fact, situation will be more interesting if we add even conventional parameters like electric charge and allow cosmological constant to vary, i.e. consideration of AdS Reissner–Nordstrom black hole. We are planning to study this separately in detail and report soon.

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