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Lead-time Dependent Ordering Cost Reduction and Trade-credit: A Supply Chain Model with Stochastic Demand and Rework

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Abstract*: This study focuses on an integrated vendor-buyer supply chain model where the lead-time and ordering cost reduction act dependently. The lead time demand of a product follows a normal distribution. The manufacturing process is imperfect. During production run time, a certain percentage of defective products are produced, which are immediately reworked. Trade-credit financing has been taken into consideration. The goal of this study is to minimize the joint total expected cost by providing an inter-dependent reduction strategy of lead-time and ordering cost along with the determination of the optimal values of lead-time, number of deliveries, order lot size, ordering cost, lead-time crashing cost, and the joint total expected cost. A solution algorithm and a numerical example are presented to illustrate and establish the integrated model. This model can be used in textiles, automobiles and computers industries.

***Keywords*: Lead time reduction, ordering cost reduction, stochastic demand, trade-credit.**

I. INTRODUCTION

To meet the purpose of reducing a total system cost, lead time as well as ordering cost reduction has been taken into consideration since the last few decades. Several authors have considered the reduction of lead time or ordering cost or both to develop their models such as Zhang, Liang, Yu, and Yu [1], Arkan and Hejazi [2], Yi and Sarker [3], and Das Roy [4]. In practice, ordering cost is considered as a constant (see Ben-Daya and Hariga [5], Das Roy, Sana, and Chaudhuri [6], Kim and Sarkar [7], and Das Roy ([8], [9])). But it can also be a variable. Some authors have considered lead time dependent ordering cost and discussed inter-dependent reduction policy of lead time and ordering cost. In 2001, Chen, Chang, and Ouyang [10] have framed an inventory model where they have introduced the idea of inter-dependent reductions of lead time and ordering cost while backorder price discount in a periodic review inventory model along with consideration of ordering cost reduction dependent on lead time is discussed by Ouyang, Chuang, and Lin [11]. Vijayashree and Uthayakumar [12] have introduced the concept of inter-dependent reduction policy of lead time and ordering cost in supply chain context and determined the optimal values of lead time, order lot size, and the number of shipments.

The benefits of the implementation of trade-credit policy attract researchers to include it in their studies. A number of research works have been incorporated by considering trade credit/permissible delay in payment policy. Chung and Cardenas-Barron [13] have investigated a supply chain model for deteriorating items having stock-dependent demand and two-level of trade-credit. A two-stage integrated supply chain model is presented by Pal, Sana, and Chaudhuri [14], where the demand is assumed to be influenced by price and credit period. Kim and Sarkar [7] have developed a vendor-buyer supply chain model where they have assumed trade-credit financing along with lead time and setup cost reduction and transportation discount.

Imperfect production in a production-inventory system is a common phenomenon. Many models have been developed by including imperfect production process (see Lee [15], Yoo, Kim, and Park [16], and Das Roy and Sana [17]). Jaber and Guiffrida [18] have studied imperfect production processes with the concept of reworks and process restoration interruptions while inventory models for reworkable items with backorder are presented by Das Roy, Sana, and Chaudhuri ([19], [20]). Chiu, Kuo, Chiu, and Hsieh [21] have developed an integrated vendor-buyer supply chain model for multiple products with rework consideration.

Here, a two-stage production-inventory supply chain model is discussed under trade credit, where the lead time demand of the buyer is assumed to be normally distributed. The manufacturing process generates perfect as well as defective products. The defective products are reworked. Both lead-time and ordering cost reduction are taken into consideration. It is assumed that ordering cost depends on lead time. A linear function of lead time is taken into consideration as a lead time crashing cost. The aim of this study is reducing the joint total expected cost by simultaneous reduction of lead-time and ordering cost and also find the optimal values of lead-time, number of deliveries, order lot size, ordering cost, and lead-time crashing cost by minimizing the joint total expected cost of the vendor-buyer supply chain system.

The rest of the paper has four sections. The notation and assumptions of the proposed model are given in Section II. Section III describes the mathematical model along with the solution algorithm. Numerical example and conclusion are presented in Section IV and Section V, respectively.

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II. NOTATION AND ASSUMPTIONS

A. Notation

The notations used to frame the proposed model are as follows.

- D Demand rate of items per unit time
- Q Buyer's order lot size (decision variable)
- P Production rate per unit time of the vendor
- S Setup cost per setup of the vendor
- A_0 Original ordering cost per order of the buyer
- C_v Production cost per unit of the vendor
- C_b Purchasing cost per unit of the buyer
- R_c Rework cost per unit of the vendor
- μ_v Vendor's annual stock holding cost per dollar invested in stocks
- μ_b Buyer's annual stock holding cost per dollar invested in stocks
- γ Percentage of defective products in the manufacturing lot
- σ Standard deviation of X
- X Lead-time demand
- r Buyer's reorder point
- n Number of delivery per production cycle (decision variable)
- k Safety factor
- L Length of lead time (decision variable)
- $A(L)$ Ordering cost per order of the buyer (decision variable), $0 < A(L) \leq A_0$.
- $Z(L)$ Lead time crashing cost per unit time of the buyer (decision variable)
- $J(.)$ Joint total expected cost per unit time

B. Assumptions

The assumptions of the proposed model are as follows.

1. This study assumes a single vendor and a single buyer.
2. The production process is imperfect. Defective products are reworked.
3. The reduction of lead time and ordering cost depends on one another. A linear expression is considered to express the interdependent relationship between the lead time and ordering cost reduction.
4. Shortages are not allowed.
5. Lead-time L has m mutually independent components. The i th component has a normal duration b_i and the minimum duration a_i with the crashing cost per unit time $c_i, i = 1, 2, 3, \dots, m$ such that $c_1 \leq c_2 \leq c_3 \leq \dots \leq c_m$.
6. Let $L_0 = \sum_{i=1}^{m} b_i$. L_i denotes the length of lead time with components $1, 2, 3, \dots, i$ crashed to their minimum duration. Let us consider $L_i = L_0 - \sum_{j=1}^{m}(b_j - a_j), i = 1, 2, 3, \dots, m$; and the lead time crashing cost per cycle $Z(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)$.
7. The lead time crashing cost $Z(L)$ is added in the buyer's expected total cost.

III. MATHEMATICAL MODEL

The model is formulated mathematical as follows.

A. Vendor's expected total cost

The production process of the vendor is imperfect. He manufactures nQ units and ships them to the buyer into n shipments each of size Q in a single cycle of length nQ/D. Let γ% of the production lot size nQ is defective. These defective items are reworked at a cost R_c per unit item. The relevant costs of the vendor per unit time are as follows.

Set up cost $= \frac{SD}{nQ}$.

Holding cost $= \frac{\mu_v C_v Q}{2}\left\{n\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right\}$ (see Das Roy [4]).

Rework cost $= R_c \gamma D$.

Opportunity interest loss $= C_b i_m DM$.

The expected total cost of the vendor per unit time is

$$TV(Q, n) = \text{setup cost} + \text{holding cost} + \text{rework cost} + \text{opportunity interest loss}$$

$$= \frac{SD}{nQ} + \frac{\mu_v C_v Q}{2}\left\{n\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right\} + R_c \gamma D + C_b i_m DM \quad (1)$$

B. Buyer's expected total cost

The lead time demand X is stochastic. It follows normal distribution with mean DL and standard deviation $\sigma\sqrt{L}$. The reorder point of the retailer is $r = DL + k\sigma\sqrt{L}$, where k is the safety factor. The lead time and ordering cost reduction depends on each other. Suppose the relation between the reduction of lead time and ordering cost is (Chen, Chang, and Ouyang [10], Vijayashree and Uthayakumar [12]) as follows

$$\frac{L_0 - L}{L_0} = \delta\left(\frac{A_0 - A}{A_0}\right)$$

where $\delta > 0$ is a scaling parameter which describes the linear relationship between the percentage of reduction in lead time and ordering cost. Therefore, the ordering cost is

$$A(L) = \alpha + \beta \ln L \quad (2)$$

where $\alpha = \left(1 - \frac{1}{\delta}\right) A_0$ and $\beta = \frac{A_0}{\delta L_0}$.

The cycle length of the buyer is Q/D and the relevant costs of the buyer per unit time are as follows.

Ordering cost $= \frac{A(L)D}{Q} = \frac{(\alpha + \beta \ln L)D}{Q}$.

Holding cost $= \mu_b C_b\left(\frac{Q}{2} + r - DL\right)$.

Lead time crashing cost $= \frac{D}{Q} Z(L)$

Interest earn $= C_s i_e D^2 M^2 / 2Q$.

Interest charged $= C_b i_c (Q - DM)^2 / 2Q$.

The expected total cost of the buyer per unit time is

$$TB(Q, L, n) = \text{ordering cost} + \text{holding cost} + \text{lead time crashing cost} + \text{interest charged} - \text{interest earn}$$

$$= \frac{(\alpha + \beta \ln L)D}{Q} + \mu_b C_b\left(\frac{Q}{2} + k\sigma\sqrt{L}\right) + \frac{D}{Q}Z(L) + \frac{C_b i_c (Q - DM)^2}{2Q} - \frac{C_s i_e D^2 M^2}{2Q} \quad (3)$$

C. Joint total expected cost

The joint total expected cost per unit time is

$$J(Q, L, n) = TV(Q, n) + TB(Q, L, n)$$

$$= \frac{D}{Q}\left\{(\alpha + \beta \ln L) + \frac{S}{n} + Z(L) + \frac{DM^2}{2}(C_b i_c - C_s i_e)\right\} + \frac{Q}{2}(H(n) + C_b i_c) + \mu_b C_b k\sigma\sqrt{L} + R_c \gamma D + C_b DM(i_m - i_c) \quad (4)$$

where $H(n) = \mu_b C_b + \mu_v C_v\left\{n\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right\}$.

Theorem 1. The joint total expected cost attains a global minimum at the optimal solution Q^* for a fixed value of n and given $L \in [L_i, L_{i-1}]$.

Proof. The first and second partial order derivatives of $J(Q, L, n)$ with respect to Q and L $L \in [L_i, L_{i-1}]$ are

$$\frac{\partial J}{\partial Q} = -\frac{D}{Q^2}\left\{(\alpha + \beta \ln L) + \frac{S}{n} + Z(L) + \frac{DM^2}{2}(C_b i_c - C_s i_e)\right\} + \frac{1}{2}(H(n) + C_b i_c) \quad (5)$$

$$\frac{\partial J}{\partial L} = \frac{\beta D}{Q} - \frac{Dc_i}{Q} + \frac{1}{2\sqrt{L}}\mu_b C_b k\sigma \quad (6)$$

$$\frac{\partial^2 J}{\partial Q^2} = \frac{2D}{Q^3}\left\{(\alpha + \beta \ln L) + \frac{S}{n} + Z(L) + \frac{DM^2}{2}(C_b i_c - C_s i_e)\right\} > 0 \quad (7)$$

$$\frac{\partial^2 J}{\partial L^2} = -\frac{1}{4\sqrt{L}}\mu_b C_b k\sigma < 0 \quad (8)$$

It is clearly from equation (8) that for a fixed Q and n, $J(Q, L, n)$ is a concave function of L. If Q, L and n are fixed then the minimum value of $J(Q, L, n)$ will occur at the end point of $[L_i, L_{i-1}]$. Also, if n is fixed and $L \in [L_i, L_{i-1}]$ is given then $J(Q, L, n)$ is a convex function of Q as $\frac{\partial^2 J}{\partial Q^2} > 0$. Thus, $J(Q, L, n)$ attains a global minimum at the optimal solution Q^*.

Hence the proof. ■

To get the solution for Q, equating equation (5) equals to zero which gives

$$Q = \sqrt{\frac{2D\left\{(\alpha + \beta \ln L) + \frac{S}{n} + Z(L) + \frac{DM^2}{2}(C_b i_c - C_s i_e)\right\}}{H(n) + C_b i_c}} \quad (9)$$

The solution procedure to find the optimal values of (Q, L, n) and $J(Q, L, n)$ are given below.

Solution Algorithm 1:

Step 1. Set $n = 1$.

Step 2. For every $L \in [L_i, L_{i-1}], i = 1, 2, 3, \ldots, m$ perform Steps (2a) –(2b).

Step 2a. Evaluate Q_i from Equation (9).

Step 2b. Utilize the value of Q_i in Equation (4) to get $J(Q_i^*, L_i, n)$.

Step 3. If $J(Q_i^*, L_i^*, n) = \text{Min}\,[J(Q_i^*, L_i, n)]$, then (Q_i^*, L_i^*, n) is the optimal solution for fixed n.

Step 4. Set $n = n + 1$ and repeat Steps (2) – (3) to find $J(Q_n^*, L_n^*, n)$.

Step 5. If $J(Q_n^*, L_n^*, n) \le J(Q_{n-1}^*, L_{n-1}^*, n-1)$, then go to Step 4, otherwise go to Step 6.

Step 6. Set $J(Q_n^*, L_n^*, n) = J(Q_{n-1}^*, L_{n-1}^*, n-1)$. Then the optimal solution is (Q_n^*, L_n^*, n^*) and the minimum value is $J(Q_n^*, L_n^*, n^*)$.

IV. NUMERICAL RESULT AND DISCUSSION

A. Numerical result

In this subsection, a suitable example is provided to illustrate and establish the model.

Example 1. Let us consider a set of parameters values most of which are taken from Vijayashree and Uthayakumar [12]. $D = 1000$ units/year, $P = 3200$ units/year, $S = \$400$ /setup, $A_0 = \$25$ /order, $C_v = \$20$ /unit, $C_b = \$25$/unit, $R_c = \$3$ /unit, $C_s = \$30$/unit, $\mu_v = \$0.2$ /unit/year, $\mu_b = \$0.2$ /unit/year, $\sigma = 7$ units/week, $k = 2.33$, $L_0 = 8$ weeks, $\gamma = 0.05$, $\delta = 5$,

$i_e = i_v = \$0.02$/unit, $i_c = \$0.06$/unit, $M = 0.1$ year and the lead time of the buyer has three components which are shown in Table I.

Table I. Lead time components with data.

Lead time component i	Normal duration b_i(days)	Minimum duration a_i(days)	Unit crashing cost c_i (days)
1	20	6	0.1
2	20	6	1.2
3	16	9	5.0

The solution procedures and the optimal solution for Example 1 are given in Table II and Table III respectively.

Table II. The solution procedure for Example 1.

L (weeks)	$Z(L)$ ($)	$A(L)$ ($)	n	Q (units)	J ($)
8	0	25	1	332	3240
			2	209	2855
			3	156	2737
			4	127	2695
			5	**108**	**2686**
			6	94	2693
			7	84	2710
6	1.4	23.75	1	332	3159
			2	209	2774
			3	156	2656
			4	127	2615
			5	**108**	**2606**
			6	94	2613
			7	84	2641
4	18.2	22.50	1	338	3108
			2	216	2750
			3	164	2656
			4	**134**	**2637**
			5	115	2648
			6	102	2674
			7	91	2709
3	53.2	21.88	1	351	3150
			2	230	2846
			3	**179**	**2799**
			4	149	2820
			5	130	2869
			6	116	2931
			7	106	2998

Table III. Optimal solution for Example 1.

L (weeks)	$Z(L)$ ($)	$A(L)$ ($)	n	Q (units)	J ($)
8	0	25	5	108	2686
6*	**1.4***	**23.75***	**5***	**108***	**2606***
4	18.2	22.50	4	134	2637
3	53.2	21.88	3	179	2799

Note: * - optimal solution

Table III shows that the optimal values of lead time $L^* = 6$ weeks, number of deliveries $n^* = 5$, order lot size $Q^* = 108$ units, lead time crashing cost $Z(L^*) = \$1.4$, ordering cost $A(L^*) = \$23.75$, and the optimum joint total expected cost $J^* = \$2606$.

B. Discussion

Our aim is to obtain the minimum joint total expected cost of the integrated system so that both of the members: the vendor and the buyer of the supply chain system are benefited. The solution of Example 1, which is shown in Table III, in boldface is the optimal solution because all the other values of the joint total expected costs (J) in Table III are greater than $\$2606$. This happens because

1) when $L = 8$ weeks, then the value of $J = \$2686$. Here the value of lead time crashing cost $Z(L)$ is zero, but the presence higher ordering cost $A(L)$ increases the value of joint total expected cost (J).
2) If $L = 4$ weeks and $L = 3$ weeks, the values of $J = \$2637$ and $J = \$2799$, respectively. In both of the cases, the values of the ordering costs $A(L)$ are comparatively low; but the lead time crashing costs $Z(L)$ are very high. Also, the order lot sizes are larger than 108 units. As a consequence, the joint total expected costs (J) have become higher.

V. CONCLUSION

In this article, a single vendor and a single buyer integrated supply chain model is investigated for reworkable items. This study considers lead-time and ordering cost reduction under trade-credit financing. It assumes that the reduction of lead-time and ordering cost depends on each other, and the percentage of lead time and ordering cost reduction follows a linear relation. The purpose of this study is to determine the optimal values of the decision variables: lead-time, number of deliveries, and order lot size by minimizing joint total expected cost. The contribution of the paper is to present an inter-dependent lead-time and ordering cost reduction policy under trade-credit for an imperfect production-inventory supply chain model. This model may be extended by considering shortages and different types of demand pattern.

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