Integrated Supply Chain Model with Setup Cost Reduction, Exponential Lead Time Crashing Cost, Rework and Uncertain Demand

Monami Das Roy

Assistant Professor, Department of Mathematics, Haldia Government College, Vidyasagar University, Purba, Medinipur, 721657, India. monamidasroy@gmail.com

Abstract - **The key purpose of any supply chain is to reduce the total system cost. Reductions of some cost parameters are helpful in this direction. The present paper deals with such a supply chain system that has included some reduction strategies. This model investigates a two stage single vendor single buyer supply chain model for a single type of product. The lead time demand follows normal distribution. During the manufacturing process, a number of defective items are generated together with perfect items. These defective items are immediately sent for rework. Lead time and setup cost reductions are taken into consideration. An exponential lead time crashing cost is assumed to reduce the lead time. The objective of this study is to minimize the joint total expected cost by reducing lead time and setup cost together with the determination of the optimal values of setup cost, order quantity, lead time and the joint total expected cost. A suitable solution algorithm is developed to determine the optimal solutions. A numerical result and some graphical representations are provided to establish the model.**

Keywords* - *Supply chain, setup cost reduction, exponential lead time crashing cost, imperfect production, rework, uncertain demand.

I. Introduction

In recent years, supply chain system receives great research attention as it helps to reduce system costs and business risks. Many activities are taken into account to reduce integrated inventory costs. Lead time and set up cost reductions are two of them. In early studies, inventory models are developed with a fixed set up cost ([1], [2], [3]). Now-a-days, researchers have realized the importance of set up cost reduction. Therefore, they have included it in their studies. The concept of setup cost reduction is introduced by Porteus [4]. He has studied an inventory model with setup cost reduction and process quality improvement. The reorder point of an inventory model is determined by Ouyang and Chang [5]. They have considered controllable lead time and setup cost. Freimer, Thomas and Tyworth [6] have presented an economic production quantity (EPQ) model with defective items where they have assumed process quality improvement and setup cost reduction. The learning effect on setup cost reduction is discussed by Pan and Lo [7]. Uthayakumar and Priyan [8] have investigated an integrated single vendor single buyer supply chain model where they have included the concept of controllable setup cost and lead time under service level constraint. They have also considered permissible delay in payments in their study.

In traditional inventory models, the lead time is considered to be zero. But it can be constant or variable. Ben-Daya and Hariga [9] have analyzed an integrated inventory model with a stochastic demand. They have considered variable lead time in their study while inventory model with a variable lead time dependent procurement cost is discussed by Chandra and Grabis [10]. Glock [11] has presented lead time reduction strategies in a two stage supply chain system for stochastic demand. He has considered lot-size dependent lead time while a single-vendor multi-buyer supply chain model with a controllable lead time and service level constraints is proposed by Jha and Shanker [12]. Vijayashree and Uthayakumar [13] have framed an integrated supply chain model including an exponential lead time crashing cost and investment for quality improvement.

In reality, it is hardly possible that a manufacturing system runs without any disturbances and produces only good quality items. Several authors have studied imperfect production processes ([14], [15], [16]). The imperfect items can be treated in different ways. Rework is one of them. Cardenas-Barron [17] has analyzed an economic production quantity (EPQ) model for a single-stage manufacturing system where he has considered rework of items and planned backorders. A single producer and single buyer supply chain system is discussed by Das Roy, Sana and Chaudhuri [2] including imperfect production process and rework. Pal, Sana and Chaudhuri [18] have investigated a three layer supply chain for reworkable items. The effect of imperfect production with rework is also discussed by

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Buscher and Lindner [19], Das Roy, Sana and Chaudhuri [20] and Chiu, Kuo, Chiu and Hsieh [21].

In this paper, an integrated vendor-buyer supply chain model is discussed for single type of items. Reduction in setup cost and lead time is taken into account. The capital investment for set up cost reduction is a logarithmic function while an exponential function of lead time is considered as lead time crash cost. The demand during the lead time is stochastic. Imperfect production is taken into consideration. Defective items are produced together with perfect items. These defective items are sent for rework. The aim of this research work is to reduce the joint total expected cost by reducing setup cost and lead time, and also obtain the optimal values of the decision variables.

The paper is split into five sections. Introduction is given in Section I. Section II contains the notation the assumptions of the model. Mathematical formulation and solution of the model are depicted in Section III. Section IV provides the numerical result while the conclusion of the whole study is presented in Section V.

II. Notation And Assumptions

A. Notation

The notations used to develop the proposed model are:

- Q order quantity of the buyer in units (decision variable)
- D demand rate in units per unit time
- P production rate in units per unit time
- n number of shipments from the vendor to the buyer in one production cycle, a positive integer
- S vendor's setup cost per setup (decision variable)
- S_0 vendor's initial setup cost per setup
- A buyer's ordering cost per order
- C_v unit production cost
- C_b unit purchase cost
- C_r unit rework cost
- r_v vendor's annual inventory holding cost per dollar invested in stocks
- r_b buyer's annual inventory holding cost per dollar invested in stocks
- L length of lead time (decision variable)
- L_N normal duration of lead time
- L_M minimum duration of lead time
- α percentage of defective items in a production lot
- $C(L)$ lead time crashing cost
- Z joint total expected cost

B. Assumptions

The assumptions of the model are as follows.

1. The model is developed for a single vendor and a single buyer.
2. The production process is imperfect. Both types of items perfect and imperfect are produced.
3. Rework of defective items is taken into consideration.
4. Lead time is a decision variable.
5. Lead time demand follows normal distribution.
6. Both lead time and setup cost are reduced.
7. An exponential function of lead time is taken as a lead time crashing cost while a logarithmic function of setup cost is considered as capital investment to reduce setup cost.
8. Shortages are not allowed.

III. Mathematical Formulation And Solution Of The Model

Suppose, the buyer orders a lot of size nQ. There is an agreement between the vendor and the buyer that the buyer will receive the whole order into n shipments where n is a known constant. The vendor produces nQ with a finite production rate P $(P > D)$ and delivers them over n times each of lot size Q. The expected cycle length of the vendor is $\frac{nQ}{D}$. During production run, a certain percentage of defective items are produced. Let α is the percentage of defective item in the whole produced lot. These defective items are immediately send for rework and restored in the main inventory.

The relevant expected costs of the vendor per unit time are as follows.

Setup cost is $= \frac{SD}{nQ}$.

This model considers the expression of the average inventory for the vendor as uses by Vijayashree and Uthayakumar's [13] that is

$$= \frac{Q}{2}\left\{n\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right\}$$

Holding cost

$$= \frac{r_v C_v Q}{2}\left\{n\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right\}$$

Let, α % of the produced items are defective. Therefore, the amount of defective items is αnQ. Rework cost is

$$= C_r \alpha nQ.\frac{D}{nQ} = C_r\alpha D$$

The total expected cost of the vendor per unit time is

$V(Q,S) = \text{setup cost} + \text{holding cost} + \text{rework cost}$

$$= \frac{SD}{nQ} + \frac{r_v C_v Q}{2}\left\{n\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right\} + C_r\alpha D \quad \ldots (1)$$

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Investment for setup cost Reduction

Suppose I_v is an investment for setup cost reduction and it is as follows.

$$I_v = U ln\left(\frac{S_0}{S}\right) \text{ for } 0 < S \le S_0$$

where $U = \frac{1}{\gamma}$, γ is the percentage decrease in S per dollar increase in I_v.

Now, the total expected cost of the vendor per unit time is

$$V(Q,S) = \beta U ln\left(\frac{S_0}{S}\right) + \frac{SD}{nQ} + C_r\alpha D + \frac{r_v C_v Q}{2}\left\{n\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right\} \quad \ldots (2)$$

where $\beta =$ annual fractional cost of the capital investment.

The expected cycle length of the buyer is $\frac{Q}{D}$. The relevant expected costs of the buyer per unit time are as follows.

Ordering cost is $= \frac{AD}{Q}$

The lead time demand follows a normal distribution with mean DL standard deviation $\sigma\sqrt{L}$. The inventory is continuously reviewed and the buyer places the order when the on hand inventory reaches to the reorder point R where $R = DL + k\sigma\sqrt{L}$ where k is a safety factor and σ is the standard deviation. The safety stock is $R - DL$. The average inventory of the buyer over a cycle is $\left(\frac{Q}{2} + \text{safety stock}\right)$.

Holding cost $= r_b C_b\left(\frac{Q}{2} + R - DL\right)$

If the buyer do not wish to add extra cost to control the lead time, he should get his items at exactly normal lead time (L_N) and crashing cost is zero. Here, it is assumed that the buyer added crashing cost to control the delivery lead time. Therefore, the buyer's lead time L should be within this interval $L_M \le L < L_N$. The lead-time crashing cost per order is considered to be an exponential function of L and is defined as

$$C(L) = \begin{cases} 0, & \text{if } L = L_N \\ e^{\frac{\mu}{L}}, & \text{if } L_M \le L < L_N \end{cases}$$

where μ is a positive constant.

The lead-time crashing cost per unit time is $= \frac{D}{Q}C(L)$.

Thus, the total expected cost of the buyer per unit time is

$$B(Q,L) = \text{ordering cost} + \text{holding cost} + \text{lead time crashing cost}$$

$$= \frac{AD}{Q} + r_b C_b\left(\frac{Q}{2} + R - DL\right) + \frac{D}{Q}C(L) \quad \ldots (3)$$

The joint total expected cost of the vendor and the buyer per unit time is

$$Z(Q,S,L) = V(Q,S) + B(Q,L)$$
$$= C_r\alpha D + \beta U(lnS_0 - lnS) + \frac{D}{Q}\left(A + \frac{S}{n} + e^{\frac{\mu}{L}}\right) + \frac{GQ}{2} + r_b C_b k\sigma\sqrt{L} \quad \ldots (4)$$

where

$$G = r_b C_b + r_v C_v\left\{n\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right\}$$

The problem can be stated as

Min $Z(Q,S,L)$

Subject to $0 < S \le S_0$

To find the solution of the above non-linear programming problem, relax the constraint $0 < S \le S_0$ and follow the classical optimization techniques as follow.

The first order derivatives of Z with respect to Q, S and L are

$$\frac{\partial Z}{\partial Q} = -\frac{D}{Q^2}\left(A + \frac{S}{n} + e^{\frac{\mu}{L}}\right) + \frac{G}{2} \quad \ldots (5)$$

$$\frac{\partial Z}{\partial S} = -\frac{\beta U}{S} + \frac{D}{nQ} \quad \ldots (6)$$

$$\frac{\partial Z}{\partial L} = -\frac{D\mu}{Q}L^{-2}e^{\frac{\mu}{L}} + \frac{r_b C_b k\sigma}{2\sqrt{L}} \quad \ldots (7)$$

For a given value of $L \in [L_M, L_N]$, setting equation (5) and (6) equal to zero which gives

$$Q^* = \sqrt{\frac{2D\left(A + \frac{S}{n} + e^{\frac{\mu}{L}}\right)}{G}} \quad \ldots (8)$$

and $$S^* = \frac{n\beta U}{D}Q \quad \ldots (9)$$

Theorem 1. The joint total expected cost function is convex at (Q^*, S^*, L^*).

Proof. The Hessian matrix H is as follows

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$$H=\begin{pmatrix}\frac{\partial^2 Z}{\partial Q^2} & \frac{\partial^2 Z}{\partial Q\partial S} & \frac{\partial^2 Z}{\partial Q\partial L}\\ \frac{\partial^2 Z}{\partial S\partial Q} & \frac{\partial^2 Z}{\partial S^2} & \frac{\partial^2 Z}{\partial S\partial L}\\ \frac{\partial^2 Z}{\partial L\partial Q} & \frac{\partial^2 Z}{\partial L\partial S} & \frac{\partial^2 Z}{\partial L^2}\end{pmatrix}$$

where

$$\frac{\partial^2 Z}{\partial Q^2}=\frac{2D}{Q^3}\left(A+\frac{S}{n}+e^{\frac{\mu}{L}}\right)>0$$

$$\frac{\partial^2 Z}{\partial S^2}=\frac{\beta U}{S^2}>0$$

$$\frac{\partial^2 Z}{\partial L^2}=\frac{De^{\frac{\mu}{L}}}{QL^4}\left(2\mu L+\mu^2\right)-\frac{r_b C_b k\sigma}{4L^{3/2}}>0$$

$$\frac{\partial^2 Z}{\partial Q\partial S}=-\frac{D}{nQ^2}=\frac{\partial^2 Z}{\partial S\partial Q}$$

$$\frac{\partial^2 Z}{\partial Q\partial L}=\frac{D\mu e^{\frac{\mu}{L}}}{Q^2L^2}=\frac{\partial^2 Z}{\partial L\partial Q}$$

$$\frac{\partial^2 Z}{\partial S\partial L}=0=\frac{\partial^2 Z}{\partial L\partial S}$$

The first order principal minor of $|H|$ at (Q^*,S^*,L^*) is

$$|H_{11}|=\frac{2D}{Q^{*3}}\left(A+\frac{S^*}{n}+e^{\frac{\mu}{L^*}}\right)>0$$

The second order principal minor of $|H|$ at (Q^*,S^*,L^*) is

$$|H_{22}|=\frac{2D\beta U}{Q^{*3}S^{*2}}\left(A+\frac{S^*}{n}+e^{\frac{\mu}{L^*}}\right)-\left(\frac{D}{nQ^{*2}}\right)^2$$

$$=\frac{D}{n^2Q^{*4}S^{*2}}\left[2n^2\beta UQ^*\left(A+\frac{S^*}{n}+e^{\frac{\mu}{L^*}}\right)-DS^{*2}\right]>0$$

since $2n^2\beta UQ^*\left(A+\frac{S^*}{n}+e^{\frac{\mu}{L^*}}\right)>DS^{*2}$

The third order principal minor of $|H|$ at (Q^*,S^*,L^*) is

$$|H_{33}|=\left[\frac{De^{\frac{\mu}{L^*}}}{Q^*L^{*4}}\left(2\mu L^*+\mu^2\right)-\frac{r_b C_b k\sigma}{4L^{*3/2}}\right]|H_{22}|-\frac{\beta U}{S^{*2}}\left(\frac{D\mu e^{\frac{\mu}{L^*}}}{Q^{*2}L^{*2}}\right)^2$$

$$=\frac{D}{n^2Q^{*4}S^{*2}L^{*4}}\left[\begin{matrix}\left\{\frac{De^{\frac{\mu}{L^*}}}{Q^*}\left(2\mu L^*+\mu^2\right)-\frac{r_b C_b k\sigma L^{*\frac{5}{2}}}{4}\right\}\\ \times\left\{2n^2\beta UQ^*\left(A+\frac{S^*}{n}+e^{\frac{\mu}{L^*}}\right)-DS^{*2}\right\}-Dn^2\beta U\mu^2e^{\frac{2\mu}{L^*}}\end{matrix}\right]>0$$

Since

$$\left\{\frac{De^{\frac{\mu}{L^*}}}{Q^*}\left(2\mu L^*+\mu^2\right)-\frac{r_b C_b k\sigma L^{*\frac{5}{2}}}{4}\right\}\left\{2n^2\beta UQ^*\left(A+\frac{S^*}{n}+e^{\frac{\mu}{L^*}}\right)-DS^{*2}\right\}>Dn^2\beta U\mu^2e^{\frac{2\mu}{L^*}}.$$

All the principal minors of the Hessian matrix H are positive. Therefore, the Hessian matrix is positive definite and the joint total expected cost function is convex at (Q^*,S^*,L^*). Hence the proof. ■

Solution Algorithm

Step 1. For all integer values of L in the interval $[L_M,L_N]$ perform step (1.1) to (1.7).

Step 1.1. Set $S=0$.

Step 1.2. Put S in equation (8) to get Q.

Step 1.3. Use Q in equation (9) to obtain S.

Step 1.4. Repeat Step (1.1) – (1.3) until no change occurs in the value of Q and S. Denote these values by (Q^*,S^*).

Step 1.5. If $S^*<S_0$, then the solution is optimal for given $L\in[L_M,L_N]$. Denote the solution by (Q^*,S^*).

Step 1.6. If $S^*\geq S_0$, then set $S^*=S_0$ and utilize equation (8) to get new Q^*. Repeat Step (1.1) – (1.4).

Step 1.7. Find $Z(Q^*,S^*,L^*)$ by using Q^*,S^* in equation (4).

Step 2. If $Z(Q^*,S^*,L^*)=Min\ Z(Q^*,S^*,L)$, then (Q^*,S^*,L^*) is the optimal solution.

IV. Numerical Result

Example 1. Let us consider a supply chain model in which the buyer's lead time demand follows normal distribution. The parameters values in suitable units are as follows. The demand of items $D=600$ units/year, production rate $P=2000$ units/year, initial setup cost of the vendor i.e. $S_0=\$1500$/setup, ordering cost $A=\$200$/order, the production cost of the vendor i.e. $C_v=\$70$/unit, the purchasing cost of the buyer i.e. $C_b=\$100$/unit, rework cost $C_r=\$5$/unit, the annual inventory holding cost of the vendor $r_v=\$0.2$/unit/year, the annual inventory holding cost of the buyer i.e. $r_b=\$0.2$/unit/year, percentage of defective items $\alpha=0.05$, vendor's annual fractional cost of the capital investment i.e. $\beta=0.1$/dollar/year, $U=18000$, safety factor $k=1.31$, number of shipments $n=3$, standard deviation $\sigma=7$ unit/week, the minimum duration of lead time i.e. $L_M=1$ week, the normal duration of lead time i.e. $L_N=10$ weeks and the lead time crashing cost is

$$C(L)=\begin{cases}0, & \text{if } L=6\\ e^{\frac{\mu}{L}}, & \text{if } 1\leq L<6\end{cases}\quad \text{where } \mu=5.$$

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The results obtained with the help of solution algorithm stated in Section III are given in Table 1.

Table 1. Optimal solution for different values of lead time

L =1			L =2			L =3		
Q	S	Z	Q	S	Z	Q	S	Z
146	1314	3983.54	**127**	**1143**	**3907.99**	126	1134	3957.31

Table 1. Continued

L =4			L =5			L =6		
Q	S	Z	Q	S	Z	Q	S	Z
125	1125	4021.62	125	1125	4061.21	125	1125	4098.35

From Table 1, it is clearly observed that the joint total expected cost i.e. $Z = \$3907.99$ is the minimum when the lead time $L = 2$ weeks. Thus, the optimal value of lead time $L^* = 2$ weeks, order quantity $Q^* = 127$ units, setup cost $S^* = \$1143$ and the optimum joint total expected cost i.e., $Z^* = \$3907.99$.

Mathematica 8.0 software is used to obtain the optimal results for Example 1 and check the optimality condition that is stated in Theorem 1. Microsoft Excel is used to draw the following graphs (see Figure 1, 2 and 3).

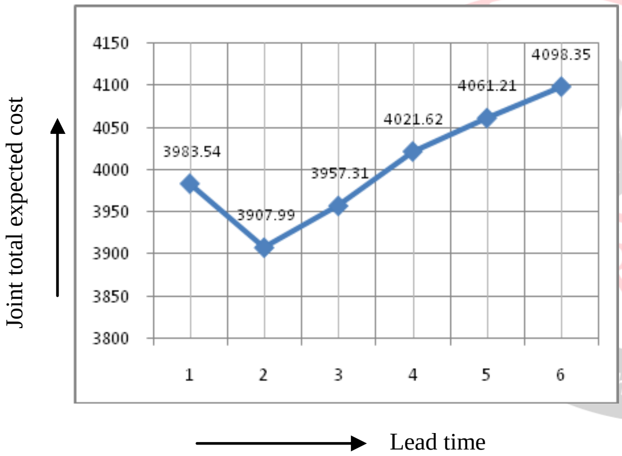

Figure 1. Lead time versus Joint total expected cost for Example 1.

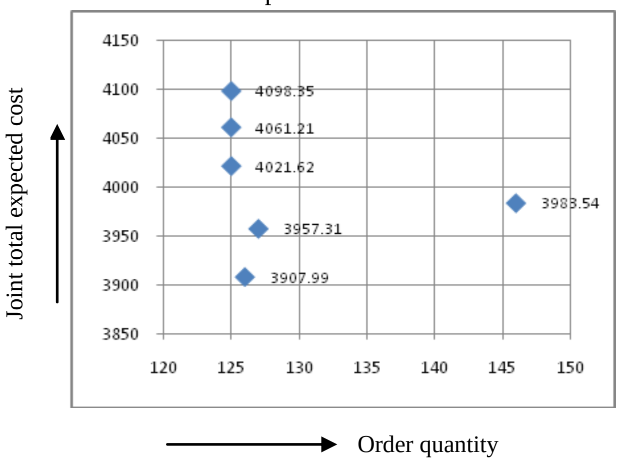

Figure 2. Order quantity versus Joint total expected cost for Example 1.

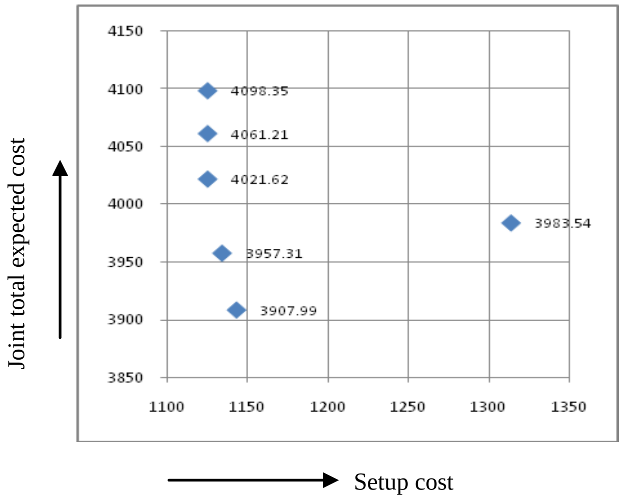

Figure 3. Setup cost versus Joint total expected cost for Example 1.

V. Conclusion

In this paper, a vendor-buyer supply chain model is incorporated for a single type of items. During the production run, both types of products, perfect and defective are produced. Defective items are immediately sent for rework. The lead time demand follows normal distribution. The reduction of lead time and setup cost is also assumed. An exponential lead time crashing cost is considered for lead time reduction. The contribution of this study is to develop a proper mathematical model and solution procedure to minimize the joint total expected cost and determine the optimal values of setup cost, order quantity of the buyer, lead time and also the joint total expected cost. Moreover, a numerical example is also provided to establish the model.

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