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Quasi stable black holes and their implications

Aloke Kumar Sinha

Ramakrishna Mission Vivekananda Educational and Research Institute, Belur Math, India
Haldia Government College, West Bengal, India

E-mail: akshooghly@gmail.com

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Abstract
We had already derived the criteria for thermal stability of charged rotating quantum black holes, for horizon areas that are large relative to the Planck area. We also had extended it for black holes with arbitrary number of hairs in arbitrary dimensional spacetime. We found that most of the criteria for thermal stability are satisfied even by some unstable black holes, although they do not satisfy all the criteria. These black holes are called 'Quasi Stable' black holes. We have already calculated thermal fluctuations and correlations among hairs of a stable quantum black hole. In this paper, we extend this work for quasi stable quantum black holes. We get the interesting result that quasi stable black holes have finite fluctuations and correlations for some of its hairs, although the black hole will ultimately radiate away due to Hawking radiation. We have also shown that quasi stable black holes may decay in a slower rate in comparision to unstable black holes.

Keywords: black hole thermodynamics, quasi stability of black hole, black hole radiation, loop quantum gravity

1. Introduction

Semiclassical analysis of black hole thermodynamics lacks from the fact that it treats spacetime as classical entity [1–5]. So, complete analysis for black hole thermodynamics has to be studied, from a perspective that relies on a definite proposal for quantum spacetime (like loop quantum gravity [6, 7]). A consistent understanding of *quantum* black hole entropy has been obtained through loop quantum gravity [8, 9], where not only has the Bekenstein–Hawking area law been retrieved for macroscopic (astrophysical) black holes, but a whole slew of corrections to it, due to quantum spacetime fluctuations have been derived as well [10–15], with the leading correction being logarithmic in area with the coefficient $-3/2$.

Thermal behaviour of a quantum black hole depends on all its hairs [16]. Using the idea of thermal holography [17, 18] and the saddle point approximation, the canonical partition function is evaluated corresponding to the horizon, retaining Gaussian thermal fluctuations. This has been generalised first for charged rotating black holes [19] and then that has been extended for black holes with arbitrary number of hairs in any spacetime dimension [16]. We have got a set of inequalities as criteria for thermal stability of a quantum black hole, immersed in a thermal bath. It is found that some black holes satisfy most of these criteria, but not all. These black holes are called 'Quasi Stable' black holes.

We have earlier calculated thermal fluctuations and correlations among hairs of a generic thermally stable quantum black hole. We then have considered AdS black holes as examples [20]. We have found that these fluctuations are finite and can be expressed as ratio of sub determinants of Hessian marix [20] that are necessarily positive for a stable black hole. We have also seen that some of the sub determinants are positive even for a quasi stable black hole. This hints to the fact that quasi stable black holes may have finite fluctuations for some of its hairs. With this motivation and previous knowledge [16, 19, 20], thermal fluctuations and correlations among the hairs of a large quasi stable black hole are calculated. It is found that our intuition is right i.e. some of these fluctuations and correlations are finite, although the black hole is ultimately unstable under Hawking radiation. We have also found that quasi stability can reduce the rate of decay of the black hole under Hawking radiation.

The paper is organized as follows: the physical meaning of the stability criteria are made clear in section 2. In the next section, procedure of detailed calculation for thermal fluctuations and correlations among various hairs are done for a general quasi stable quantum black hole. This section is followed by the section containing the calculations for some quasi stable black holes as examples. In the succeeding section, we have described the relationship between quasi stability and decay rate of a quasi stable black hole under Hawking radiation. The last section contains a brief summary and outlook.

2. Black hole stability and thermal fluctuation

In semiclassical analysis of black hole thermodynamics, black holes are still treated classically and its boundary is represented by the event horizon [3, 4]. Partition function is evaluated there by allowing the fluctuations of metric of the black hole. But in a full-fledged theory of quantum gravity, we do not require any global knowledge of black hole spacetime through its metric. Infact black holes at equilibrium are there represented by isolated horizons, which are internal boundaries of spacetime. Hence mass of a black hole can be defined locally on this horizon [21, 22]. It has been shown [16, 19] that partition function of a black hole can be derived interms of its boundary partition function only. It has also been shown there that partition function of a black hole, in a theory of quantum gravity, turns out to be given in terms of fluctuations of its parameters that contribute to its mass.

Consider a black hole immersed in a heat bath, at some (inverse) temperature β, with which it can exchange energy and all its 'n' hairs(charges). The equilibrium configuration of the black hole is given by the saddle point $(\bar{A}, \bar{C}^1, \ldots, \bar{C}^n)$ in the $(n+1)$ dimensional space of integration over area and n charges, where $\bar{A}$ is horizon area A at equilibrium and $\bar{C}^i$ is the charge C^i at equilibrium. The grand canonical partition function is calculated for fluctuations $a = (A - \bar{A})$, $c^i = (C^i - \bar{C}^i)$ around the saddle point. Hence the grand canonical partition function (Z_G) can be written as [19]

$$Z_{\mathrm{G}} = \exp[S(\bar{A}) - \beta M(\bar{A}, \bar{C}^1, ..., \bar{C}^n) + \beta P_i \bar{C}^i] \times \int \mathrm{d}a \left(\prod_{i=1}^{n} \int \mathrm{d}c^i\right) \exp\{-\frac{1}{2}[(\beta M_{AA} - S_{AA})a^2 + 2\sum_{i=1}^{n} \beta M_{AC^i} ac^i + \sum_{i=1}^{n}\sum_{j=1}^{n} \beta M_{C^iC^j} c^i c^j]\}, \quad (1)$$

where P_i, β are potential corresponding to charge C^i and inverse temperature of the black hole respectively. The stability criteria have been derived [16] from the above expression of partion function (Z_{G}) and are given as follows:

$$D_1 = (\beta M_{AA} - S_{AA}) > 0, D_2 = \begin{vmatrix} \beta M_{AA} - S_{AA} & \beta M_{AC^1} \\ \beta M_{AC^1} & \beta M_{C^1C^1} \end{vmatrix} > 0, \ldots, D_{n+1} = |H| > 0. \quad (2)$$

Where, $|H|$ = determinant of the real symmetric $(n+1)$ dimensional square Hessian matrix H, whose elements are $H_{11} = (\beta M_{AA} - S_{AA})$, $H_{1i} = \beta M_{AC^i}$, $H_{i+1,j+1} = \beta M_{C^iC^j}; i, j = 1, \ldots n$. Of course, (inverse) temperature β is assumed to be positive for a stable configuration.

If a black hole is totally chargeless, then positivity of D_1 will be the sole criteria for its thermal stability. For such a black hole, specific heat (C) can be expressed as, $C \equiv \mathrm{d}M/\mathrm{d}T = (S_A)^2/D_1$. Thus positivity of D_1 implies positivity of specific heat and hence the black hole is thermally stable.

For an arbitrary black hole, variation of its temperature (T) with area is given as, $\mathrm{d}T/\mathrm{d}A = D_1 \cdot M_A/(S_A)^2$. So, a black hole having negative D_1 will either grow in size indefinitely as it cools down or shrink indefinitely as it becomes hotter and hotter. Both the situations are the sign of instability.

The fluctuation for the charge C^i will be denoted as $\Delta(C^i)^2$. Similarly $\Delta(A)^2$ will denote the fluctuation in area of the black hole. The correlation between charge C^i and C^j will be denoted as $\Delta C^i C^j$. So, $\Delta C^i A$ will correspond the correlation between charge C^i and area A. We will strict to this notation[1] throughout the paper.

We will now see that $D_1, D_2, D_3, \ldots, D_{n+1}$ are related to the fluctuations of the charges and area of the black hole. For sake of simplicity, we take $n = 1$. In this case, stability criteria will be $D_1, D_2 > 0$. The fluctuation of charge C^1 and area A are give as [20],

$$\Delta A^2 = \beta M_{C^1C^1}/2D_2, \qquad \Delta(C^1)^2 = D_1/2D_2.$$

$\therefore$ Both ΔA^2 , $\Delta(C^1)^2$ decrease with the increment of D_1, D_2 and vice versa. Thus gradual decrement of D_1, D_2 will increase the fluctuations and hence entropy of the black hole will decrease [25]. So the entropy of the outside universe will increase because of Hawking radiation. When, ultimately, one of the D_1, D_2 becomes negative, Hawking radiation starts to dominate over accretion completely. As a result of this, black hole starts to decay. This analysis is true for any value of n. Thus $D_1, \cdots, D_{n+1}$ controll the entire thermodynamics of a black hole around its equilibrium.

Instability of a black hole is artifected by the fact that unbounded fluctuations take the black hole far away from the equilibrium via Hawking radiation. Quasi stable black holes are ultimately unstable. Hence one would expect that fluctuations for a quasi stable black hole would blow up and hence it would drive the system far from equilibrium. But, surprisingly,

[1] Notations, used in this paper, differ from the notations used in ref no. [20]. In that ref, $(\Delta C^i)^2$ denotes the fluctuation for charge C^i and so on.

this answer is not entirely right. We will see that fluctuations of some hairs can be finite for quasi stable black holes. This fact is really a stepping stone to understand the physical meaning of quasi stability. We will describe the mathematical formulation of fluctuations and correlations among hairs for these black holes in the next section.

3. Fluctuation theory of quasi stable black holes

We know how to calculate fluctuations and correlations for a stable black hole within the rigime of stability in parameter space [20]. In those cases, we can calculate the partition function in any basis of our convenience as the result is converging. We can thereon calculate fluctuations and correlations for various hairs of the black hole. But this procedure does not hold for quasi stable black holes as partition function is diverging. So, we have to necessarily rearrange the partition function in the diagonal basis of Hessian matrix and we then have to look for stable modes. We can calculate fluctuations only for these stable modes, although the partition function is diverging.

Now, we can rewrite the expression (1) of grand canonical partition function (Z_{G}) in the diagonal basis of Hessian matrix as,

$$Z_{\mathrm{G}} = \Big(\prod_{j=1}^{n+1}\int \mathrm{d}\underline{c}^{j}\Big)\exp\{-\frac{1}{2}[D_1(\underline{c}^1)^2 + \frac{D_2}{D_1}(\underline{c}^2)^2 + \ldots + \frac{D_{n+1}}{D_n}(\underline{c}^{n+1})^2]\} \tag{3}$$

where the expressions of $D_1, D_2, \ldots, D_{n+1}$ are same as given in (2). The new variables $(\underline{c}^1, \ldots, \underline{c}^{n+1})$ are related to the old variables $(a, c^1, \ldots, c^n)$ by some linear transformation. The linear transformation matrix is a $(n+1)$ dimensional upper triangular square matrix and hence it has unit determinant. The elements of this transformation matrix are functions of the elements of the Hessian matrix H.

If atleast one of $D_1, \frac{D_2}{D_1}, \ldots, \frac{D_{n+1}}{D_n}$ is neagative, then Z_{G} blows up. This means that the black hole can not be stable. Z_{G} diverges in all the directions $c^1, c^2, \ldots, c^{n+1}$ i.e. diverges maximally when $D_1, \frac{D_2}{D_1}, \ldots, \frac{D_{n+1}}{D_n}$ are all negative individually. This implies,

$$\begin{aligned} &D_1, D_3, D_5, \ldots < 0 \\ &D_2, D_4, D_6, \ldots > 0. \end{aligned} \tag{4}$$

This shows that maximally diverging partition function corresponds to quasi stable black holes.

On the other hand, if the black hole is totally unstable i.e. $D_1, D_2, D_3, \ldots, D_{n+1}$ are simultaneously negative then Z_{G} diverges only in the direction of $\underline{c}^1$ i.e. Z_{G} is minimally diverging.

It was shown [16] that a black hole with n charges has to satisfy $(n+1)$ conditions to become thermally stable. But it was also proven that [19] electrically charged, rotating black hole has to satisfy seven conditions to prove its stability. Actually three of these seven conditions are independent, rest depend on those three conditions. This fact is true only for stable black holes, not for quasi stable black holes. Let us consider $n = 1$ case to understand this important issue.

$n = 1$ implies that black hole has only one charge C^1. Thus the stability criteria is given as [19] $\beta M_{C^1C^1}, D_1, D_2 > 0$.

If D_1 and D_2 are positive, then $\beta M_{C^1C^1}$ has to be positive and hence stability criteria can also be written as, $D_1 > 0$ and $D_2 > 0$ [16]. But if D_1 and D_2 are negative, then sign of $\beta M_{C^1C^1}$ is free. Thus a black hole of n charges, having $D_1, D_2, \cdots, D_{n+1} < 0$, can be quasi stable as well. To ensure it, one has to check the positivity of determinants of all $(2^{n+1} - 1)$ submatrices of Hessian matrix H (including itself). Even if one of them is positive, then black hole is quasi stable.

We will now discuss the formulation of calculation for the fluctuations of quasi stable black holes. For sake of simplicity as well as practical consideration, we will take 'n' equals to 2. But all the results hold equally for arbitary n. For $n = 2$, we can write down the partition function as,

$$Z_{\mathrm{G}} = \int\int\int \mathrm{d}\underline{a}\mathrm{d}\underline{c}^1\mathrm{d}\underline{c}^2 \ \cdot \exp\Big(-\frac{1}{2}\big(D_1\underline{a}^2 + \frac{D_2}{D_1}(\underline{c}^1)^2 + \frac{D_3}{D_2}(\underline{c}^2)^2\big)\Big) \tag{5}$$

where,

$$\begin{pmatrix}\underline{a}\\ \underline{c}^1\\ \underline{c}^2\end{pmatrix} = \begin{pmatrix}1 & \frac{d}{x} & \frac{f}{x}\\ 0 & 1 & \frac{e-df/x}{((xb-\mathrm{d}^2)/x)}\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}a\\ c^1\\ c^2\end{pmatrix}. \tag{6}$$

Instead of denoting the new basis as $(\underline{c}^1, \underline{c}^2, \underline{c}^3)$, we are denoting it as $(\underline{a}, \underline{c}^2, \underline{c}^3)$.

Various terms are given as,

$$x = \beta M_{AA} - S_{AA}, b = \beta M_{C^1C^1}, d = \beta M_{AC^1}, c = \beta M_{C^2C^2}, e = \beta M_{C^1C^2}, f = \beta M_{AC^2}$$

$$\therefore D_1 = x, D_2 = (xb - \mathrm{d}^2), D_3 = \Big(x(bc - e^2) - d(cd - ef) + f(de - bf)\Big)$$

Now consider a quasi stable black hole, having $D_1, D_2, D_3 < 0$. This consideration is extremely interesting as upto this point we can not distinguish quasi stable black holes from completely unstable black holes. We will see later how can we really distinguish quasi stable black holes from completely unstable black holes and move on to calculate fluctuations for quasi stable black holes.

The fluctuation for the charge $(\underline{C}^2)^2$ [2] is given as,

$$\Delta(\underline{C}^2)^2 = \frac{\int\int\int \mathrm{d}\underline{a}\mathrm{d}\underline{c}^1\mathrm{d}\underline{c}^2 \ \cdot (\underline{c}^2)^2 \cdot \exp\Big(-\frac{1}{2}\big(D_1\underline{a}^2 + \frac{D_2}{D_1}(\underline{c}^1)^2 + \frac{D_3}{D_2}(\underline{c}^2)^2\big)\Big)}{\int\int\int \mathrm{d}\underline{a}\mathrm{d}\underline{c}^1\mathrm{d}\underline{c}^2 \ \cdot \exp\Big(-\frac{1}{2}\big(D_1\underline{a}^2 + \frac{D_2}{D_1}(\underline{c}^1)^2 + \frac{D_3}{D_2}(\underline{c}^2)^2\big)\Big)}. \tag{7}$$

This is a converging intergral and is equal to $D_2/2D_3$. Relation (6) implies that $c^2 = \underline{c}^2$ and hence $\Delta(C^2)^2 = \Delta(\underline{C}^2)^2$.

Similary we can calculate the fluctuation for the charge$(\underline{C}^1)$ and is given as, $\Delta(\underline{C}^1)^2 = D_1/2D_2$. Ofcourse $\underline{C}^1$ is not a physical charge, but it is a combination of physical charges C^1, C^2 and that combination can be extracted out from the relation (6).

It is clear from relation (6) that we can choose appropriate tranformation matrix such that $\underline{c}^1 = c^1$. In that case, $\Delta(C^1)^2$ would be given, from symmetry argument, as,

[2] Fluctuation of charge C^2 is denoted as c^2. Similarly, we are defining charge $\underline{C}^2$ as whose fluctuation is $\underline{c}^2$. Therefore new charge $\underline{C}^1$ can be defined in same spirit.

$$\Delta(C^1)^2 = \frac{\left(\beta M_{AA} - S_{AA}\right) \cdot \beta M_{C^2C^2} - (\beta M_{AC^2})^2}{2D_3}.$$

We will now assume, for example, that these quasi stable black holes (i.e. $D_1, D_2, D_3 < 0$) have the properties, namely, $M_{C^1C^1} > 0$ and $(M_{C^1C^1}M_{C^2C^2} - (M_{C^1C^2})^2) > 0$. We are considering quasi stable black holes of this kind as we have already tested the stability criteria of them [16, 19]. Anyway, our formalism does hold in general. In this case, we will denote the new basis as $(\tilde{a}, \tilde{c^1}, \tilde{c^2})$ and is related to the old basis as,

$$\begin{pmatrix} \tilde{c^1} \\ \tilde{c^2} \\ \tilde{a} \end{pmatrix} = \begin{pmatrix} 1 & \frac{d}{b} & \frac{e}{b} \\ 0 & 1 & \frac{f-de/b}{(c-e^2/b)} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c^1 \\ c^2 \\ a \end{pmatrix}. \tag{8}$$

Where, (b, c, d, e, f) are same as before.

We can now express the partition function in the new basis and can calculate fluctuations, exactly like the previous situation. The fluctuations are calculated as, $\Delta(\tilde{C}^1)^2 = 1/2b$, $\Delta(\tilde{C}^2)^2 = \frac{b}{2(bc-e^2)}$ and ΔA^2 blows up.

Now, $\underline{C}^1$ is linear combination of physical charges C^1, C^2 and hence $\Delta(\underline{C}^1)^2$ is a linear combination of fluctuations and correlations between C^1 and C^2. Similarly $\Delta(\tilde{C}^1)^2$ and $\Delta(\tilde{C}^2)^2$ are same among C^1, C^2 and A. This fact, along with the equations (6) and (8), implies that $\Delta C^1 C^2$ is finite, while $\Delta C^1 A$ and $\Delta C^2 A$ are diverging. These results can be viewed as follows:

In the three dimensional space of fluctuation (a, c^1, c^2), fluctuations are converging in the particular two dimensional subspace, consisting of c^1 and c^2. That is why the black hole is stable in this subspace. But departure from this subspace makes the black hole unstable with large fluctuations in other directions. This makes the black hole quasi stable as a whole.

4. Examples of fluctuations for quasi stable black holes

It can be easily shown that asymptotically flat Schwarzschild black hole (AFSBH) is thermally unstable as $D_1 (= \beta M_{AA} - S_{AA})$ is always negative. It has already been shown that asymptotically flat Kerr–Newman black hole (AFKNBH) is also thermally unstable [19]. But the terms M_{QQ}, $(M_{QQ}M_{JJ} - (M_{QJ})^2)$ are positive throughout the parameter space for AFKNBH. Infact all the stability criteria, except the positivity of $|H|$, hold in some region of parameter space(A, Q, J). Thus comparing AFSBH and AFKNBH, we can conclude that addition of charge Q and angular momentum J tend to stabilize the black hole, although ultimately remain unsuccessful. So, black holes having charge and angular momentum are the viable candidates to be quasi stable.

We have studied black holes having traditional hairs like electric charge, angular momentum [19] and also have studied black holes with non traditional hairs like magnetic charge, two independent angular momentum etc [16]. Among these examples, it was found that Kerr–Sen black hole and Asymptotically flat Kerr–Newman black holes have some similarities in structure. Thus Kerr–Sen black hole is extremely interesing for the purpose of investigation of quasi stable black holes and their fluctuations. So, we will start with Kerr–Sen black hole in details.

4.1. Kerr–Sen black hole

Kerr–Sen black hole is also known as asymptotically flat string theoretic black hole [23]. We have already proven that this type of black hole can not be thermally stable under Hawking radiation [19]. The dependence of its mass on its charge, area and angular momentum can be read from [23] and is given as,

$$M^2 = \frac{A}{16\pi} + \frac{Q^2}{2} + \frac{4\pi J^2}{A}. \tag{9}$$

One can easily show from the above expression that M_{QQ}, M_{JJ} and $(M_{QQ}M_{JJ} - (M_{JQ})^2)$ are always positive. Hence Kerr–Sen black hole is quasi stable under Hawking radiation.

With resemblance to the general analysis of fluctuation theory, we assume $C^1 \equiv Q$ and $C^2 \equiv J$. We have already seen that positivity of temperature implies $\frac{J}{A} < \frac{1}{8\pi}$. We will now assume that both $\frac{J}{A}$ and $\frac{Q^2}{A}$ are sufficiently smaller than unity. Ofcourse we can calculate fluctuations in any regime. We choose this particular regime as this is the far extremal limit of AFKNBH and hence we can check the similarities in fluctuations of these two black holes. We will calculate the fluctuations, but only the leading order contribution, in the limit $\frac{Q^2}{A}, \frac{J}{A} \ll 1$. In this limit eqn no. (9) can be approximated as, $M \approx \frac{A^{1/2}}{4\pi^{1/2}} + \frac{\pi^{1/2}Q^2}{A^{1/2}} + \frac{8\pi^{3/2}J^2}{A^{3/2}} - \frac{2\pi^{3/2}Q^4}{A^{3/2}}$. Ofcourse this is the leading order approximation. Infact we will now on consider leading order values only. Using this approximated form of mass and the formalism described in the last section, we can easily calculate ΔQ^2, ΔJ^2 and ΔQJ and they are given as,

$$\Delta Q^2 \approx \frac{A_P}{8\pi}, \quad \Delta J^2 \approx \frac{A_P A}{64\pi^2} \quad \text{and} \quad \Delta QJ \approx -\frac{3\pi^2 A_P QJ}{A}$$

It is extremely intersting to note that leading order fluctuation of charge and angular momentum are independent of charge and angular momentum, like stable AdS black holes. Infact for charge, this value is a constant. Again, $\frac{|\Delta QJ|}{QJ} \approx \frac{3\pi^2 A_P}{A}$ and is tiny small fraction. More importantly this measure of correlation does not depend on charge (Q) and angular momentum (J). These values show that Kerr–Sen black hole may not be stable under Hawking radiation, but they are completely stable in (q, j) subspace of the whole fluctuation space.

4.2. Kerr–Newman black hole

It has already been proven that Kerr–Newman black hole can not be thermally stable under Hawking radiation [19]. The Smarr formula expresses mass of the black hole as function of its area, charge and angular momentum and it is read as [24]

$$M^2 = \frac{A}{16\pi} + \frac{\pi}{A}(4J^2 + Q^4) + \frac{Q^2}{2}. \tag{10}$$

It is easy to check that M_{QQ}, M_{JJ} and $\left(M_{JJ}M_{QQ} - (M_{JQ})^2\right)$ are always positive and hence Kerr–Newman black hole is quasi stable under Hawking radiation, exactly like Kerr–Sen black hole.

Now we have alrady seen [19] that positivity of temperature, for this black hole, implies that $\frac{J}{A} < \frac{1}{8\pi}$ and $\frac{Q^2}{A} < \frac{1}{4\pi}$. We will now consider KN black holes that are far extremal i.e. $\frac{J}{A} \ll \frac{1}{8\pi}$ and $\frac{Q^2}{A} \ll \frac{1}{4\pi}$. We choose this regime to calculate various fluctuations only for computational simplicity, but same formalism will hold in any regime. We are here interested to calculate the leading order contributions only. In the far extremal limit, the mass of the black hole (10) can be approximated as, $M \approx \frac{A^{1/2}}{4\pi^{1/2}} + \frac{\pi^{1/2}Q^2}{A^{1/2}} + \frac{8\pi^{3/2}J^2}{A^{3/2}}$. This approximated form of mass enables us to calculate the leading order values of $\Delta J^2, \Delta Q^2, \Delta QJ$ and they are exactly same as that of far extremal KS black hole with small charge and angular momentum. This implies that in far extremal limit KN black hole behaves like far extremal KS black hole with small charge, in thermodynamic sense.

Thus we see that a black may not be completely stable under Hawking radiation i.e. quasi stable, but it can still have various interesting thermodynamic properties, like a fully stable black hole.

5. Quasi stability and hawking decay

Thermal stability criteria are obtained in form of a series of inequalities. Quasi stable black holes satisfy some of these criteria simultaneously but not all. Thus quasi stable black holes possess features of both stable and unstable black holes. We have shown that quasi stable black holes do have small fluctuations for some of its hairs like a stable black hole. Stable black holes do not decay under Hawking radiation. Fluctuations of all the hairs of a stable black hole is very small and hence the black hole remains stable under Hawking radiation. Now all the fluctuations are not small for a quasi stable black hole and hence it ultimately decays under Hawking radiation. But some of its fluctuations are small and hence a quasi stable black hole is expected to show some resistance in its decay process. Hence an overall delay may occur during the full process of decay of the quasi stable black hole.

The decay of a black hole is approximately governed by Stefan–Boltzmann law as the profile of black hole radiation is approximately equal to that of a black body. So, luminosity (L), defined as the power radiated per unit surface area, is proportional to the fourth power of its temperature (T) i.e. $L \propto T^4$. So the variation of luminosity with the area of black hole is given as,

$$\begin{aligned} \frac{\mathrm{d}L}{\mathrm{d}A} &\propto \frac{\mathrm{d}T}{\mathrm{d}A} \\ &\propto D_1 \\ &= \beta M_{AA} - S_{AA}. \end{aligned} \tag{11}$$

Now, D_1 is negative for a unstable black hole, e.g. asymptotically flat schwarzschild black hole and hence luminosity gradually increases with the decay of the black hole. But if D_1 is positive for some quasi stable black hole, then luminosity gradually decreses with the decay of this black hole. Thus the black hole tries to resist the decay process and hence delay occurs in the process. Now, we will consider the examples of two quasi stable black holes and will show that D_1 is positive for them in some region of parameter space.

The mass of a AFKN black hole is given as, $M^2 = \frac{A}{16\pi} + \frac{\pi}{A}(4J^2 + Q^4) + \frac{Q^2}{2}$. Thus positivity of temperature ($T = \frac{M_A}{S_A}$) implies $A^2 > 16\pi^2(4J^2 + Q^4)$. We also find that positivity of D_1 implies $96\pi^2(4J^2 + Q^4) > A^2$. So, a real AFKN black hole has positive D_1 in the regime $96\pi^2(4J^2 + Q^4) > A^2 > 16\pi^2(4J^2 + Q^4)$. This implies that this black hole has a 'window' in parameter space such that D_1 is positive there.

Similarly the mass of quasi stable Kerr–Sen black hole is given as, $M^2 = \frac{A}{16\pi} + \frac{4\pi J^2}{A} + \frac{Q^2}{2}$. We can show like previous case that D_1 is positive in the 'window' $8\pi J < A < 8\sqrt{6}\pi J$. Infact this 'window' is obtained for small value of charge Q and actually the 'window' will be broadened for large value of Q.

From the above two examples, we find that quasi stable black holes may not lie within the 'window' at early stage due to large area and hence negativity of D_1 makes decay process gradually rapider as it progresses. After becoming appropriately smaller, black hole enters in the 'window' and this makes D_1 positive. Hence the black hole starts to show its repulsion to support the decay process. Thus delay occurs and consequently life time of the black hole increases.

6. Discussion

Quasi stable black holes, like unstable black holes, have diverging partition function with large fluctuations for some of its hairs. They ultimately decay under Hawking radiation. But simultaneouly these black holes have finite fluctuations for some of its hairs. They show resistance in the decay process of Hawking radiation. This feature is somewhat like a stable black hole. Thus quasi stable black holes have this duality property. It is to be noted that our formalism holds for macroscopic black holes i.e. black holes whose area is larger than Planck area. But close to the end state, the area of a black hole is comparable to Planck area and hence we have to solve the complicated Hamiltonian constraint. We can nevertheless say that there would be some remnant of a black hole at the end state, with a minimum area according to the theory like LQG. This fact has been mentioned in [26] where it is concluded that these remnants could form component of dark matter as well. In this sense, our analysis may have some impacts on the dark matter physics.

In ordinary thermodynamics, people are interested to calculate thermal fluctuations of various macroscopic parameters of a thermodynamical system as these quantities are related to physically measurable quantities e.g. fluctuation in energy measures the specific heat of a system. We, earlier, had derived [16, 19] the criteria for thermal stability as certain mathematical inequalities. But the correponding physical meanings were not clear at that time. But the meanings are made clear in this paper. The inequalities of stability criteria are directly related to fluctuation and correlation functions. We had made [19] certain assumptions regarding the nature of quantum spacetime and LQG is a theory which supports them. But our assumptions are so general that any theory of quantum gravity would respect those. Hence the notion of quasi stability is also expected to be extremely generic in any theory of quantum gravity. We have studied KS and KN black holes as example of quasi stable black holes. We found that in far extremal limit, fluctuations for certain parameters of these black holes match with that of stable AdS black holes. Now, the AdS/CFT correspondence tells that asymptotically AdS black hole is dual to a strongly coupled gauge theory at finite temperature [27–30]. It is possible to analyze the strongly correlated condensed matter physics using AdS/CFT correspondence. Thus our calculations for quasi stable black hole may have some imprints to condensed matter physics as well.

ORCID iDs

Aloke Kumar Sinha https://orcid.org/0000-0003-4367-3339

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