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Thermal fluctuations and correlations among hairs of a stable quantum black hole: Some examples

Aloke Kumar Sinha
Ramakrishna Mission Vivekananda Educational and Research Institute, Belur Math, India
Haldia Government College, West Bengal, India
akshooghly@gmail.com

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We have already derived the criteria for thermal stability of charged rotating quantum black holes, for horizon areas that are large relative to the Planck area. The derivation is done by using results of loop quantum gravity and equilibrium statistical mechanics of the grand canonical ensemble. We have also showed that in four-dimensional spacetime, quantum AdS Kerr–Newman black hole and asymptotically AdS dyonic black hole with electric and magnetic charge are thermally stable within certain range of its parameters. In this paper, the expectation values of fluctuations and correlations among horizon area, electric charge and angular momentum (magnetic charge) of these black holes are calculated within their range of stability. Interestingly, it is found that leading order fluctuations of electric charge and angular momentum (magnetic charge), in large horizon area limit, are independent of the values of electric charge and angular momentum (magnetic charge) at equilibrium.

Keywords: Quantum black hole; black hole thermodynamics; thermal stability.

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1. Introduction

It has been shown semiclassically that non-extremal, asymptotically flat black holes are thermally unstable under Hawking radiation, with negative specific heat.[1] This motivated the study of thermal stability of black holes from the perspective of quantum spacetime (like Loop Quantum Gravity,[2,3]). A consistent understanding of quantum black hole entropy has been obtained through Loop Quantum Gravity,[4,5] where a whole slew of corrections to the Bekenstein–Hawking area law, due to quantum spacetime fluctuations, have been derived,[6–11] with the leading correction being logarithmic in area with the coefficient $-3/2$.

In general relativity, a black hole is characterized by its mass (M), charge (Q) and angular momentum (J). So, it is expected that thermal behavior of a quantum black hole will depend on all of these parameters. The simplest case of vanishing charge and angular momentum had been investigated a long time ago[12–14] and that has been generalized, via the idea of thermal holography,[15,16] and the saddle point approximation to evaluate the canonical partition function corresponding to the horizon, retaining Gaussian thermal fluctuations. This body of work has been generalized recently[17] for charged rotating black holes. It is shown that anti-de Sitter (AdS) Kerr–Newman black hole (for a certain range of its parameters) is thermally stable. In fact, the conditions for thermal stability of a macroscopic quantum black hole with arbitrary number of hairs in arbitrary spacetime dimension has already been derived too.[18] It turns out that asymptotically AdS dyonic black holes with electric and magnetic charge are also thermally stable within some region of its parameter space.

In this paper, using previous knowledge,[17,18] thermal fluctuations and correlations among all the hairs i.e. electric charge, horizon area and angular momentum (magnetic charge) are calculated. These are calculated in the limit of large horizon area.

The paper is organized as follows. In Sec. 2, detailed calculation of thermal fluctuations and correlations are done for AdS Kerr–Newman black hole. In the next section, same thing has been done for asymptotically AdS dyonic black holes with electric and magnetic charge. A brief summary and outlook have been given in the very next section. We end with an Appendix. It contains the derivations of various results, used in the main body of the paper and necessary formulas from Refs. 17 and 18, used in this paper.

2. Thermal Fluctuation and Correlation among Hairs of AdS Kerr–Newman Black Hole

The expectation value of fluctuation of any quantity is the standard deviation of that quantity. It is a statistical measure of deviation for any distribution. The knowledge of probability theory and the expression of grand canonical partition function (A.2) together give the standard deviation of charge (Q) as,

$$(\Delta Q)^2 = \frac{\begin{array}{c}\int da\,dq\,dj\,q^2\exp\Big\{-\frac{\beta}{2}\Big[\Big(M_{AA}-\frac{S_{AA}}{\beta}\Big)a^2+(M_{QQ})q^2+(2M_{AQ})aq\\ +(M_{JJ})j^2+(2M_{AJ})aj+(2M_{QJ})qj\Big]\Big\}\end{array}}{\begin{array}{c}\int da\,dq\,dj\,\exp\Big\{-\frac{\beta}{2}\Big[\Big(M_{AA}-\frac{S_{AA}}{\beta}\Big)a^2+(M_{QQ})q^2+(2M_{AQ})aq\\ +(M_{JJ})j^2+(2M_{AJ})aj+(2M_{QJ})qj\Big]\Big\}\end{array}}, \qquad (1)$$

where ΔQ is the standard deviation of black hole charge. Similarly, ΔA and ΔJ are defined for horizon area and angular momentum of the black hole.

The correlation function between charge (Q) and angular momentum (J) is denoted as ΔQJ and is defined as

$$\Delta QJ = \frac{\begin{array}{c}\int da\,dq\,dj\,qj\exp\Big\{-\frac{\beta}{2}\Big[(M_{AA}-\frac{S_{AA}}{\beta})a^2+(M_{QQ})q^2+(2M_{AQ})aq \\ +(M_{JJ})j^2+(2M_{AJ})aj+(2M_{QJ})qj\Big]\Big\}\end{array}}{\begin{array}{c}\int da\,dq\,dj\exp\Big\{-\frac{\beta}{2}\Big[\Big(M_{AA}-\frac{S_{AA}}{\beta}\Big)a^2+(M_{QQ})q^2+(2M_{AQ})aq \\ +(M_{JJ})j^2+(2M_{AJ})aj+(2M_{QJ})qj\Big]\Big\}\end{array}}\,. \tag{2}$$

Similarly, ΔQA and ΔJA are defined for the black hole.

The expressions (1), (A.2) and (A.4) together give,

$$(\Delta Q)^2 = -\frac{2}{\beta}\cdot\frac{1}{Z_G}\cdot\frac{\partial Z_G}{\partial M_{QQ}} = \frac{1}{|H|}\cdot\frac{\partial|H|}{\partial(\beta M_{QQ})}\,, \tag{3}$$

where $|H|$ = determinant of Hessian matrix (H).

Similarly, $(\Delta A)^2$, $(\Delta J)^2$, ΔQA, ΔJA, ΔQJ are defined by taking partial derivatives with respect to $\big(M_{AA}-\frac{S_{AA}}{\beta}\big)$, M_{JJ}, M_{QA}, M_{JA} and M_{QJ}, respectively, i.e.

$$(\Delta A)^2 = -\frac{2}{\beta}\cdot\frac{1}{Z_G}\cdot\frac{\partial Z_G}{\partial(M_{AA}-\frac{S_{AA}}{\beta})} = \frac{1}{|H|}\cdot\frac{\partial|H|}{\partial(\beta M_{AA}-S_{AA})}\,, \tag{4}$$

$$(\Delta J)^2 = -\frac{2}{\beta}\cdot\frac{1}{Z_G}\cdot\frac{\partial Z_G}{\partial M_{JJ}} = \frac{1}{|H|}\cdot\frac{\partial|H|}{\partial(\beta M_{JJ})}\,, \tag{5}$$

$$\Delta QA = -\frac{1}{\beta}\cdot\frac{1}{Z_G}\cdot\frac{\partial Z_G}{\partial M_{QA}} = \frac{1}{2|H|}\cdot\frac{\partial|H|}{\partial(\beta M_{QA})}\,, \tag{6}$$

$$\Delta JA = -\frac{1}{\beta}\cdot\frac{1}{Z_G}\cdot\frac{\partial Z_G}{\partial M_{JA}} = \frac{1}{2|H|}\cdot\frac{\partial|H|}{\partial(\beta M_{JA})}\,, \tag{7}$$

$$\Delta QJ = -\frac{1}{\beta}\cdot\frac{1}{Z_G}\cdot\frac{\partial Z_G}{\partial M_{QJ}} = \frac{1}{2|H|}\cdot\frac{\partial|H|}{\partial(\beta M_{QJ})}\,. \tag{8}$$

Equations (3) and (A.8) together give

$$(\Delta Q)^2 = \frac{1}{|H|}\cdot((\beta M_{AA}-S_{AA})\cdot\beta M_{JJ}-(\beta M_{AJ})^2)\,, \tag{9}$$

where $|H|$ is the determinant of the Hessian matrix (H).

Similarly, Eqs. (A.8) and (4)–(8) together give, respectively,

$$(\Delta A)^2 = \frac{1}{|H|}\cdot(\beta^2(M_{QQ}M_{JJ}-(M_{JQ})^2))\,, \tag{10}$$

$$(\Delta J)^2 = \frac{1}{|H|}\cdot((\beta M_{AA}-S_{AA})\cdot\beta M_{QQ}-(\beta M_{AQ})^2)\,, \tag{11}$$

$$\Delta QA = \frac{1}{|H|}\cdot(\beta^2(M_{JQ}M_{AJ}-M_{AQ}M_{JJ}))\,, \tag{12}$$

$$\Delta JA = \frac{1}{|H|} \cdot (\beta^2(M_{JQ}M_{AQ} - M_{AJ}M_{QQ}))\,, \tag{13}$$

$$\Delta QJ = \frac{1}{|H|} \cdot (\beta^2 M_{AQ}M_{AJ} - \beta M_{JQ}(\beta M_{AA} - S_{AA}))\,. \tag{14}$$

The AdS Kerr–Newman black hole is given in Boyer–Lindquist coordinates as

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Sigma}d\phi\right)^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left(\frac{r^2+a^2}{\Sigma}d\phi - a\,dt\right)^2$$
$$+ \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2\,, \tag{15}$$

where $\Sigma = 1 - \frac{a^2}{l^2}$, $\Delta_r = (r^2+a^2)\big(1+\frac{r^2}{l^2}\big) - 2Mr + Q^2$, $\Delta_\theta = 1 - \frac{a^2 cos^2\theta}{l^2}$, $\rho^2 = r^2 + a^2\cos^2\theta$, $a = \frac{J}{M}$.

The generalized Smarr formula for the AdS Kerr–Newman black hole is given as[20]

$$M^2 = \frac{A}{16\pi} + \frac{\pi}{A}(4J^2+Q^4) + \frac{Q^2}{2} + \frac{J^2}{l^2} + \frac{A}{8\pi l^2}\left(Q^2 + \frac{A}{4\pi} + \frac{A^2}{32\pi^2 l^2}\right), \tag{16}$$

where the cosmological constant (Λ) is defined in terms of a cosmic length parameter as $\Lambda = -1/l^2$.

As before, our interest is in astrophysical (macroscopic) charged, rotating black holes whose horizon area is very large compared to the Planck area. In Ref. 17, it is shown that AdS Kerr–Newman black holes are stable if $A^2 \gg (4J^2+Q^4)$ and $A \gg l^2$. So, we can approximate (16) as follows:

$$M \approx \frac{A^{3/2}}{16\pi^{3/2}l^2} + \frac{A^{1/2}}{4\pi^{1/2}} + \frac{\pi^{1/2}Q^2}{A^{1/2}} + \frac{8\pi^{3/2}J^2}{A^{3/2}}\,. \tag{17}$$

The detailed calculation is given in Appendix A.

Equations (A.9) and (A.10) together give

$$(\Delta Q)^2 \approx \frac{3A_pA}{16\pi^2 l^2}\,. \tag{18}$$

The detailed calculation is given in Appendix A.

Similarly, Eqs. (A.9) and (10)–(14) together give

$$(\Delta A)^2 \approx 8A_pA\,, \tag{19}$$

$$(\Delta J)^2 \approx \frac{3A_pA^2}{128\pi^3 l^2}\,, \tag{20}$$

$$\Delta QA \approx 4A_PQ\,, \tag{21}$$

$$\Delta JA \approx 12A_PJ\,, \tag{22}$$

$$\Delta QJ \approx \frac{6A_PJQ}{A}\,. \tag{23}$$

Of course, last six expressions are only the leading order terms in large horizon area limit.

It is extremely interesting to note that $\langle(\Delta J)^2\rangle$ and $\langle(\Delta Q)^2\rangle$ are independent of J and Q, respectively. This implies that there are finite amount of fluctuations of charge and angular momentum even for an almost neutral, non-rotating macroscopic AdS black hole, i.e. AdS black hole with $J, Q \to 0$.

The measure of area fluctuation is given as

$$\frac{\Delta A}{A} \approx \sqrt{\frac{8A_P}{A}}\,. \tag{24}$$

Similarly, measure of other fluctuations and correlations are given as

$$\frac{\Delta Q}{Q} \approx \sqrt{\frac{3}{16\pi^2}} \cdot \frac{\sqrt{A_P A}}{Ql}\,, \tag{25}$$

$$\frac{\Delta J}{J} \approx \sqrt{\frac{3}{128\pi^3}} \cdot \frac{\sqrt{A_P A^2}}{Jl}\,, \tag{26}$$

$$\sqrt{\frac{\Delta QA}{QA}} \approx \sqrt{\frac{4A_P}{A}}\,, \tag{27}$$

$$\sqrt{\frac{\Delta QJ}{QJ}} \approx \sqrt{\frac{6A_P}{A}}\,, \tag{28}$$

$$\sqrt{\frac{\Delta AJ}{AJ}} \approx \sqrt{\frac{12A_P}{A}}\,. \tag{29}$$

Equations (24), (27)–(29) imply that all the measure of correlations and area fluctuations become infinitesimal for large black holes ($A \gg A_P$). This is the feature of fluctuation around stable equilibrium point. These are interestingly independent of charge (Q), angular momentum (J) of the black hole.

Table A.1 suggests that consideration of (17) would imply

$$\frac{A^{3/2}}{16\pi^{3/2}l^2} > \frac{\pi^{1/2}Q^2}{A^{1/2}}\,, \qquad \frac{A^{3/2}}{16\pi^{3/2}l^2} > \frac{8\pi^{3/2}J^2}{A^{3/2}}\,. \tag{30}$$

Equations (25), (26) and (30) together give

$$\frac{\Delta Q}{Q} > \sqrt{\frac{3A_P}{A}}\,, \tag{31}$$

$$\frac{\Delta J}{J} > \sqrt{\frac{3A_P}{A}}\,. \tag{32}$$

Last two expressions imply that measure of charge and angular momentum fluctuations are infinitesimal for large black holes ($A \gg A_P$). Although these expressions (31) and (32) are the lower bounds, they are independent of charge (Q) and angular momentum (J) of the black hole and eventually are zero in large black hole limit.

3. Thermal Fluctuation and Correlation among Hairs of Asymptotically AdS Dyonic Black Holes with Electric and Magnetic Charge

The four-dimensional metric for this black hole is given as[21]

$$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + R^2\,d\Omega^2\,, \tag{33}$$

where

$$\phi = \frac{\phi_3}{r^3} + \mathcal{O}(r^{-4})\,, \tag{34}$$

$$f = \frac{-\Lambda r^2}{3} + 1 - \frac{2M}{r} + \frac{Q^2 + P^2}{r^2} + \frac{\Lambda\phi_3^2}{5r^4} + \mathcal{O}(r^{-5})\,, \tag{35}$$

$$R = r - \frac{3\phi_3^2}{20r^5} + \mathcal{O}(r^{-6})\,, \tag{36}$$

$$\phi_3 = \frac{g_0}{\Lambda}\int_{r_h}^{\infty}\frac{dr}{R^2}(Q^2\exp(2g_0\phi) - P^2\exp(-2g_0\phi))\,, \tag{37}$$

where r_h, Q, P are the radius of horizon, electric charge and magnetic charge of the black hole, respectively. $\Lambda(<0)$ is the cosmological constant, g_0 is diatonic coupling constant and ϕ is the diatonic field.

It has been shown that[18] thermal stability is possible if $A \gg A_P$, l^2, Q^2, P^2.

In this limit, mass of this black hole is given as[18]

$$M \approx \frac{A^{3/2}}{48l^2\pi^{3/2}} + \frac{A^{1/2}}{4\pi^{1/2}} + \frac{\pi^{1/2}(Q^2+P^2)}{A^{1/2}}\,, \tag{38}$$

where $\Lambda = -1/l^2$, l is the cosmic length.

Now, fluctuations and correlations among hairs of this black hole can be calculated like earlier. In fact, all the formulas (3)–(14) do hold in this case with the understanding that angular momentum (J) should be replaced by magnetic charge (P).

Equations (9), (38), (A.8), (A.5) and (A.6) together give

$$(\Delta Q)^2 \approx \frac{A_p A}{16\pi^2 l^2}\,. \tag{39}$$

Similarly, other fluctuations and correlations can be calculated and are given as

$$(\Delta A)^2 \approx 8A_p A\,, \tag{40}$$

$$(\Delta P)^2 \approx \frac{A_p A}{16\pi^2 l^2}\,, \tag{41}$$

$$\Delta QA \approx 4A_P Q\,, \tag{42}$$

$$\Delta PA \approx 4A_P P\,, \tag{43}$$

$$\Delta QP \approx \frac{2A_P PQ}{A}\,. \tag{44}$$

It turns out that $\langle(\Delta P)^2\rangle$ and $\langle(\Delta Q)^2\rangle$ are independent of P and Q, respectively. This implies that large asymptotically AdS dyonic black hole with vanishing electric and magnetic charge can have finite amount of fluctuations of electric and magnetic charge.

The measure of fluctuations and correlations are given as

$$\frac{\Delta A}{A} \approx \sqrt{\frac{8A_P}{A}}\,, \tag{45}$$

$$\frac{\Delta Q}{Q} \approx \frac{1}{4\pi} \cdot \frac{\sqrt{A_P A}}{Ql}\,, \tag{46}$$

$$\frac{\Delta P}{P} \approx \frac{1}{4\pi} \cdot \frac{\sqrt{A_P A}}{Pl}\,, \tag{47}$$

$$\sqrt{\frac{\Delta QA}{QA}} \approx \sqrt{\frac{4A_P}{A}}\,, \tag{48}$$

$$\sqrt{\frac{\Delta QJ}{QJ}} \approx \sqrt{\frac{2A_P}{A}}\,, \tag{49}$$

$$\sqrt{\frac{\Delta AP}{AP}} \approx \sqrt{\frac{4A_P}{A}}\,. \tag{50}$$

Equations (45), (48)–(50) imply that measure of area fluctuation and all the correlations are independent of electric charge (Q), magnetic charge (P) and they are infinitesimal for large black holes ($A \gg A_P$).

Thermal stability of asymptotically AdS dyonic black holes with electric and magnetic charge is possible if $A \gg A_P$, l^2, Q^2, P^2. This condition along with Eqs. (46) and (47) implies that

$$\frac{\Delta Q}{Q} > \frac{1}{4\pi} \cdot \sqrt{\frac{A_P}{A}}\,, \tag{51}$$

$$\frac{\Delta P}{P} > \frac{1}{4\pi} \cdot \sqrt{\frac{A_P}{A}}\,. \tag{52}$$

So, measure of electric and magnetic charge fluctuations are infinitesimal for large black holes ($A \gg A_P$). Although last two expressions of (51) and (52) are the lower bounds, they are independent of electric and magnetic charges (Q, P) and eventually are zero in large black hole limit.

4. Summary and Discussion

The novelty of our analysis is that it is quite independent of specific classical spacetime geometries, relying as it does on quantum aspects of spacetime. The construction of the partition function used standard formulations of equilibrium statistical mechanics augmented by results from canonical quantum gravity, with extra inputs regarding the behavior of the microcanonical entropy as a function of area *beyond the Bekenstein–Hawking area law*, as for instance derived from Loop Quantum Gravity.[7] We use classical metric only as an input which gives the dependence of mass on its charges.

In large horizon area limit, it turns out that for a quantum AdS black hole, leading order fluctuations of electric charge ($(\Delta Q)^2$) and angular momentum ($(\Delta J)^2$) are independent of its charge (Q) and angular momentum (J). This implies that even a black hole with infinitesimal charge (Q) and angular momentum (J) can have finite fluctuations in respective quantities. Similar is the conclusion for the case of asymptotically AdS dyonic black holes with electric and magnetic charge. The S_{AA} term is present everywhere in the calculation. The nonvanishing contribution of this term is pure artifact of quantum fluctuation of spacetime. Thermal fluctuations are present along with this quantum fluctuation as we are considering black hole to be immersed in an extended thermal bath. So, thermal fluctuations and correlations that we have calculated, take care of quantum fluctuation of spacetime automatically. Thus, it is extremely interesting in its own merit. We choose these examples: AdS black holes as AdS/CFT correspondence tells that string theory on AdS space is dual to a conformal field theory (CFT) on the boundary of that AdS space.[22,23] It has also been shown using the AdS/CFT correspondence that the asymptotically AdS black hole is dual to a strongly coupled gauge theory at finite temperature.[24–27] It is possible to study the strongly correlated condensed-matter physics using the AdS/CFT correspondence. Holographic model of superconductors has also been constructed from black hole solutions using the AdS/CFT correspondence.[28] Hence, our results of AdS black holes may have some imprints on possible applications for the strongly correlated condensed-matters systems.

Appendix A

Consider a macroscopic black hole immersed in a heat bath, at some (inverse) temperature β, with which it can exchange energy, charge, angular momentum and all quantum hairs. The grand canonical partition function of a black hole with n charges is given as[18]

$$Z_G = \int dA \left(\prod_{i=1}^{n} \int dC^i\right) \exp(S(A) - \beta(E(A, C^1 \cdots C^n) - P_i C^i)) , \qquad \text{(A.1)}$$

where following Ref. 19, the *microcanonical* entropy of the horizon is defined by $\exp[S(A)] \equiv \frac{g(A(x),C(y_1)\cdots C(y_n))}{\frac{dA}{dx}\frac{dC^1}{dy_1}\cdots\frac{dC^n}{dy_n}}$. Here, C^i is the ith charge with corresponding potential P_i.

If we choose $n = 2$ with the identification $C^1 = Q$ and $C^2 = J$, then we will get back AdS Kerr–Newman black hole. Similarly, $n = 2$, with the identification $C^1 = Q$ and $C^2 = P$, will give back asymptotically AdS dyonic black holes with electric and magnetic charge.

For $n = 2$, the expression of grand canonical partition function would take the form,[17]

$$Z_G = \exp[S(\bar{A}) - \beta M(\bar{A}, \bar{Q}, \bar{J}) + \beta\Phi\bar{Q} + \beta\Omega\bar{J}] \int da\, dq\, dj$$
$$\times \exp\left\{-\frac{\beta}{2}\left[\left(M_{AA} - \frac{S_{AA}}{\beta}\right)a^2 + (M_{QQ})q^2 + (2M_{AQ})aq\right.\right.$$
$$\left.\left. + (M_{JJ})j^2 + (2M_{AJ})aj + (2M_{QJ})qj\right]\right\}, \tag{A.2}$$

where $M(\bar{A}, \bar{Q}, \bar{J})$ is the mass of equilibrium isolated horizon. Here, $M_{AQ} \equiv \frac{\partial^2 M}{\partial A \partial Q}|_{(\bar{A},\bar{Q},\bar{J})}$, etc.

The above expression (A.2) of grand canonical partition function can be expanded in Taylor series around saddle point.[18] The stability criteria[17,18] was derived from this expression, based on the Hessian matrix (H). The Hessian matrix, for black hole with n charges, is given as[18]

$$H = \begin{pmatrix} \beta M_{AA} - S_{AA} & \beta M_{AC^1} & \beta M_{AC^2} & \dots & \beta M_{AC^n} \\ \beta M_{AC^1} & \beta M_{C^1C^1} & \beta M_{C^1C^2} & \dots & \beta M_{C^1C^n} \\ \beta M_{AC^2} & \beta M_{C^2C^1} & \beta M_{C^2C^2} & \dots & \beta M_{C^2C^n} \\ \dots & \dots & \dots & \dots & \dots \\ \beta M_{AC^n} & \beta M_{C^nC^1} & \beta M_{C^nC^2} & \dots & \beta M_{C^nC^n} \end{pmatrix}, \tag{A.3}$$

where $\beta\left(= \frac{S_A}{M_A}\right)$ is the inverse temperature.

The Hessian matrix (H) is symmetric. So, it can be diagonalized by some orthogonal matrix. Therefore, if $\lambda_1, \dots, \lambda_{n+1}$ be $(n+1)$ eigenvalues of H, then Z_G will be given as

$$Z_G \propto \frac{1}{\sqrt{(\lambda_1 \cdot \lambda_2 \cdots \lambda_{n+1})}}$$
$$= \frac{1}{\sqrt{|H|}}, \tag{A.4}$$

where $|H|$ is the determinant of the Hessian matrix (H).

Since we are considering quantum theory of gravity, we have to consider the effect of quantum spacetime fluctuations on microcanonical entropy of isolated horizons. It has been shown that[7] the microcanonical entropy for macroscopic isolated horizons (S) in $(3+1)$-dimensional spacetime has the form

$$S = S_{\mathrm{BH}} - \frac{3}{2}\log S_{\mathrm{BH}} + \mathcal{O}(S_{\mathrm{BH}}^{-1}), \tag{A.5}$$

$$S_{\mathrm{BH}} = \frac{A}{4A_P}, \ A_P \to \text{Planck area}. \tag{A.6}$$

Therefore, Planck area (A_P) enters in the Hessian through temperature $(= \frac{1}{\beta})$ and, consequently, in the stability criteria.

Table A.1. Stable points of AdS KN black hole.

Value of $u\left(=\frac{A}{l^2}\right)$	Value of $\frac{x}{u}\left(=\frac{(J/A)}{(A/l^2)}\right)$	Value of $\frac{y}{u}\left(=\frac{(Q^2/A)}{(A/l^2)}\right)$
1	9.99×10^{-3}	8.99×10^{-2}
10^1	3.00×10^{-3}	2.60×10^{-2}
10^2	8.99×10^{-4}	1.96×10^{-2}
10^3	8.19×10^{-4}	1.89×10^{-2}
10^4	7.75×10^{-4}	1.88×10^{-2}
10^5	7.24×10^{-4}	1.87×10^{-2}

The region of parameter space for stability of AdS Kerr–Newman black hole had been shown in Ref. 17 and a sample table is given as above.

This table shows the selected six points in the $(u, \frac{x}{u}, \frac{y}{u})$ space, such that AdS KN black hole is stable in these points. This table of course shows the maximum possible values of $\frac{x}{u}$, $\frac{y}{u}$ for a given value of "u" within the region of stability.

In Ref. 17, it is mentioned that if "l" is sufficiently large then AdS KN black hole will behave like a KN black hole and it will be thermally unstable. We want to study some interesting properties for thermal fluctuation of stable AdS KN black hole. So, we have assumed that area of macroscopic black hole horizon (A) is sufficiently larger that "l^2". In Ref. 17, it is shown that AdS Kerr–Newman black holes are stable if $A^2 \gg (4J^2 + Q^4)$ and $A \gg l^2$.

Now we can write down Eq. (16) as

$$M^2 = \frac{A^3}{256\pi^3 l^4} + \frac{A^2}{32\pi^2 l^2} + A\left(\frac{Q^2}{8\pi l^2} + \frac{1}{16\pi}\right) + \left(\frac{Q^2}{2} + \frac{J^2}{l^2}\right) + \frac{1}{A}(4\pi J^2 + \pi Q^4)\,.$$

This implies that

$$M = \frac{A^{3/2}}{16\pi^{3/2} l^2}\left(1 + \frac{8\pi l^2}{A} + \frac{1}{A^2}(32\pi^2 l^2 Q^2 + 16\pi^2 l^4) + \frac{1}{A^3}(128\pi^3 l^4 Q^2 + 256\pi^3 J^2 l^2) + \frac{256\pi^3 l^4}{A^4}(4\pi J^2 + \pi Q^4)\right)^{1/2}. \quad (A.7)$$

Therefore, it is possible to approximate the last expression within the range of stability, i.e. $(A^2 \gg (4J^2 + Q^4)$ and $A \gg l^2)$ as

$$M \approx \frac{A^{3/2}}{16\pi^{3/2} l^2} + \frac{A^{1/2}}{4\pi^{1/2}} + \frac{\pi^{1/2} Q^2}{A^{1/2}} + \frac{8\pi^{3/2} J^2}{A^{3/2}}\,.$$

This is precisely Eq. (17).

Now, the determinant of the Hessian matrix (H) is given as

$$\begin{aligned} |H| &= \beta^2(\beta M_{AA} - S_{AA})(M_{QQ}M_{JJ} - M_{QJ}^2) \\ &\quad - \beta^3 M_{AQ}(M_{AQ}M_{JJ} - M_{QJ}M_{AJ}) \\ &\quad + \beta^3 M_{AJ}(M_{AQ}M_{QJ} - M_{QQ}M_{AJ})\,. \end{aligned} \quad (A.8)$$

We are interested in leading order calculation for fluctuations and correlations of AdS KN black hole. All the terms in the expression of this Hessian are calculable from Eq. (17). The leading order values of various terms are given as

$$\beta \approx \frac{8\pi^{3/2}l^2}{3A^{1/2}A_P}\,, \qquad S_{AA} = \frac{3}{2A^2}\,, \qquad M_{AA} \approx \frac{3}{64\pi^{3/2}l^2A^{1/2}}\,,$$
$$M_{JJ} \approx \frac{16\pi^{3/2}}{A^{3/2}}\,, \qquad M_{QQ} \approx \frac{2\pi^{1/2}}{A^{1/2}}\,, \tag{A.9}$$
$$M_{AJ} \approx -\frac{24\pi^{3/2}J}{A^{5/2}}\,, \qquad M_{AQ} \approx -\frac{\pi^{1/2}Q}{A^{3/2}}\,.$$

On calculation, it can be shown easily that $\beta^2(\beta M_{AA} - S_{AA})(M_{QQ}M_{JJ} - M_{AA}^2)$ is the dominating term in the expression of $|H|$ (A.8). So, Eq. (9) implies that the leading order value of $(\Delta Q)^2$ can be written as

$$(\Delta Q)^2 \approx \frac{1}{\beta M_{QQ}}\,. \tag{A.10}$$

Therefore, we get $(\Delta Q)^2 \approx \frac{3A_pA}{16\pi^2l^2}$, as given in (18).

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