Modern Physics Letters A
Vol. 33 (2018) 1850031 (11 pages)

DOI: 10.1142/S0217732318500311

Thermal stability of black holes with arbitrary hairs

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Received 25 December 2017
Revised 16 January 2018
Accepted 18 January 2018
Published 8 February 2018

We have derived the criteria for thermal stability of charged rotating black holes, for horizon areas that are large relative to the Planck area (in these dimensions). In this paper, we generalized it for black holes with arbitrary hairs. The derivation uses results of loop quantum gravity and equilibrium statistical mechanics of the grand canonical ensemble and there is no explicit use of classical spacetime geometry at all in this analysis. The assumption is that the mass of the black hole is a function of its horizon area and all the hairs. Our stability criteria are then tested in detail against some specific black holes, whose metrics provide us with explicit relations for the dependence of the mass on the area and other hairs of the black holes. This enables us to predict which of these black holes are expected to be thermally unstable under Hawking radiation.

Keywords: Quantum black hole; thermal stability; quantum gravity.

PACS Nos.: 04.70.-s, 04.70.Dy

1. Introduction

The semiclassical analysis of thermal instability of non-extremal, asymptotically flat black holes with negative specific heat[1] motivates to study the same, entirely from quantum perspective like loop quantum gravity.[2,3] A consistent understanding of quantum black hole entropy can be obtained from loop quantum gravity.[4,5] The modified Bekenstein–Hawking area law has been derived[6–11] for macroscopic (astrophysical) black holes, with the leading correction being logarithmic in area with the coefficient $-3/2$.

Classically, a black hole in general relativity is characterized by its mass (M), charge (Q) and angular momentum (J). Intuitively, therefore, we expect that thermal behavior of a black hole will depend on all of its hairs from quantum perspective. These additional hairs will change the energy function of the black hole. This will in turn make the stability criteria more complicated. It is obvious that stability criteria will now depend on the interplay among all hairs. So, it is really interesting to redo, what we have done earlier,[12] taking the contributions of additional hairs. We will do this in this paper in detail. We will assume that the mass of black hole is a function of the horizon area and all its hairs.

The idea of thermal holography,[13,14] and the saddle point approximation are used to evaluate the canonical partition function corresponding to the horizon, retaining Gaussian thermal fluctuations. This leads to a general criterion of thermal stability as a set of inequalities connecting first- and second-order area derivatives of microcanonical entropy along with first- and second-order derivatives of mass with respect to area and all the hairs. These are derived in detail in this paper. This inequality is nontrivial when the microcanonical entropy has corrections (of a particular algebraic sign) beyond the area law, as is the case for the loop quantum gravity calculation of the microcanonical entropy.[7] This procedure has recently been done for charged rotating black hole[12] and the derived stability criterion indeed "predicts" the thermal instability of asymptotically flat Kerr–Newman black holes contrasted with the thermal stability of anti-de Sitter Kerr–Newman black holes (for a range of its parameters).

The paper is organized as follows. In Sec. 2, the idea of thermal holography, along with the concept of (holographic) mass associated with horizon of a black hole is briefly reviewed and the grand canonical entropy of a large black hole, with arbitrary number of hairs is determined. In Sec. 3, the criterion for thermal stability of such black holes is determined by using saddle point approximation to evaluate the horizon partition function for Gaussian thermal fluctuations around thermal equilibrium. Here, we also compare our stability criteria with that obtained earlier as a particular case. In Sec. 4, this stability criterion is used to test various black holes, with the objective of predicting their behavior under decay due to Hawking radiation. We end in Sec. 5 with a brief summary and outlook.

2. Thermal Holography

In this section, we present a generalization of the thermal holography for rotating electrically charged radiant horizons discussed in Ref. 12, to the situation when the horizon has arbitrary number of hairs. To make this section self-contained, some overlap with Ref. 12 is inevitable.

2.1. *Mass associated with horizon*

Black holes at equilibrium are represented by isolated horizons, which are internal boundaries of spacetime. Hamiltonian evolution of this spacetime gives the first law associated with isolated horizon (b) and is given as

$$\delta E_h^t = \frac{\kappa^t}{8\pi}\delta A_h + P_i^t \delta C_h^i \,. \tag{1}$$

Here, Einstein summation convention is used, i.e. summation over repeated indice i from 1 to $n\,(=$ total number of hairs) is implied and E^t_h is the energy function associated with the horizon. κ^t, P^t_i are, respectively, the surface gravity associated with the area of horizon (A_h) and the potential corresponding to the charge (hair) C^i_h. The label "t" denotes the particular time evolution field (t^μ) associated with the spatial hypersurface chosen. E^t_h is assumed here to be a function of A_h and all C^i_h.

The advantage of the isolated (and also the radiant or *dynamical*) horizon description is that one can associate with it a mass M^t_h, related to the ADM energy of the spacetime through the relation

$$E^t_{\mathrm{ADM}} = M^t_h + E^t_{\mathrm{rad}}\,, \tag{2}$$

where E^t_{rad} is the energy associated with spacetime between the horizon and asymptopia. An isolated horizon admits $E^t_{\mathrm{rad}} \neq 0$, and hence a mass is defined *locally* on the horizon.

2.2. *Quantum geometry*

The Hilbert space of a generic quantum spacetime is given as $\mathcal{H} = \mathcal{H}_b \otimes \mathcal{H}_v$, where $b(v)$ denotes the boundary (bulk) space. A generic quantum state is thus given as

$$|\Psi\rangle = \sum_{b,v} C_{b,v}|\chi_b\rangle \otimes |\psi_v\rangle\,. \tag{3}$$

Now, the full Hamiltonian operator ($\hat{H}$), operating on $\mathcal{H}$ is given by

$$\hat{H}|\Psi\rangle = (\hat{H}_b \otimes I_v + I_b \otimes \hat{H}_v)|\Psi\rangle\,, \tag{4}$$

where, respectively, $I_b(I_v)$ are identity operators on $\mathcal{H}_b(\mathcal{H}_v)$ and $\hat{H}_b(\hat{H}_v)$ are the Hamiltonian operators on $\mathcal{H}_b(\mathcal{H}_v)$.

The first class constraints are realized on Hilbert space as annihilation constraints on physical states. The bulk Hamiltonian operator thus annihilates bulk physical states

$$\hat{H}_v|\psi_v\rangle = 0\,. \tag{5}$$

The bulk quantum spacetime is assumed to be free of any charge (hair), so that Eq. (5) is augmented by the relation

$$[\hat{H}_v - P_i\hat{C}^i_v]|\psi_v\rangle = 0\,. \tag{6}$$

2.3. *Grand canonical partition function*

Consider the black hole immersed in a heat bath, at some (inverse) temperature β, with which it can exchange energy, charge, angular momentum and all other hairs. The grand canonical partition function of the black hole is given as

$$Z_G = \mathrm{Tr}(\exp(-\beta\hat{H} + \beta P_i\hat{C}^i))\,, \tag{7}$$

where the trace is taken over all states. This definition, together with Eqs. (3), (5) and (6) yield

$$Z_G = \sum_{b,v} |C_{b,v}|^2 \langle\psi_v|\psi_v\rangle\langle\chi_b|\exp(-\beta\hat{H}_b + \beta P_i\hat{C}^i)|\chi_b\rangle$$
$$= \sum_b |C_b|^2 \langle\chi_b|\exp(-\beta\hat{H}_b + \beta P_i\hat{C}^i)|\chi_b\rangle\,, \quad (8)$$

assuming that the bulk states are normalized. The partition function thus turns out to be completely determined by the boundary states ($Z_{\rm Gb}$), i.e.

$$Z = Z_{\rm Gb} = {\rm Tr}_b \exp(-\beta\hat{H} + \beta P_i\hat{C}^i)$$
$$= \sum_{l,k_1,\ldots,k_n} g(l,k_1,\ldots,k_n)\exp\left(-\beta\left(E(A_l, C^1_{k_1},\ldots,C^n_{k_n}) - \sum_{i=1}^{n} P_i C^i_{k_i}\right)\right), \quad (9)$$

where $g(l,k_1,\ldots,k_n)$ is the degeneracy corresponding to energy $E(A_l, C^1_{k_1},\ldots,C^n_{k_n})$ and l, k_i are the quantum numbers corresponding to area and charge C^i, respectively. Here, the spectrum of the boundary Hamiltonian operator is assumed to be a function of area and all other charges of the boundary, considered here to be the horizon. Following Ref. 15, it is further assumed that these "hairs" all have a discrete spectrum. In the semiclassical limit of quantum isolated horizons of macroscopic area, they all have large eigenvalues, i.e. $(l, k_i \gg 1)$, so that, application of the Poisson resummation formula[16] gives

$$Z_G = \int dx \left(\prod_{i=1}^{n}\int dy_i\right) g(A(x), C^1(y_1),\ldots,C^n(y_n))$$
$$\times \exp\left(-\beta\left(E(A(x), C^1(y_1),\ldots,C^n(y_n)) - \sum_{i=1}^{n} P_i C^i(y_i)\right)\right), \quad (10)$$

where x, y_i are, respectively, the continuum limit of l, k_i, respectively.

Following Ref. 15, we now assume that the semiclassical spectrum of the area and charges are linear in their arguments, so that a change of variables gives, with constant Jacobian, the result

$$Z_G = \int dA \left(\prod_{i=1}^{n}\int dC^i\right) \exp(S(A) - \beta(E(A, C^1,\ldots,C^n) - P_i C^i))\,, \quad (11)$$

where, following Ref. 17, the *microcanonical* entropy of the horizon is defined by $\exp S(A) \equiv \frac{g(A(x),C(y_1),\ldots,C(y_n))}{\frac{dA}{dx}\frac{dC^1}{dy_1}\cdots\frac{dC^n}{dy_n}}$.

3. Stability Against Gaussian Fluctuations

3.1. *Saddle point approximation*

The equilibrium configuration of black hole is given by the saddle point $\bar{A}$, $\bar{C}^i$ in the $(n+1)$-dimensional space of integration over area and n charges. This configuration is identified with an isolated horizon, as already mentioned. The idea now is to examine the grand canonical partition function for fluctuations $a = (A - \bar{A}), c^i = (C^i - \bar{C}^i)$ around the saddle point, in order to determine the stability of the equilibrium isolated horizon under Hawking radiation. We restrict our attention to Gaussian fluctuations. Taylor expanding Eq. (11) about the saddle point, yields

$$Z_G = \exp[S(\bar{A}) - \beta M(\bar{A}, \bar{C}^1, \ldots, \bar{C}^n) + \beta P_i \bar{C}^i] \int dA \left(\prod_{i=1}^{n} \int dC^i \right) \times \exp\left\{ -\frac{1}{2} \left[(\beta M_{AA} - S_{AA}) a^2 + 2 \sum_{i=1}^{n} \beta M_{AC^i} a c^i + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta M_{C^i C^j} c^i c^j \right] \right\}, \tag{12}$$

where $M(\bar{A}, \bar{C}^1, \ldots, \bar{C}^n)$ is the mass of equilibrium isolated horizon. Here, $M_{AC^i} \equiv \partial^2 M / \partial A \partial C^i |_{(\bar{A}, \bar{C}^1, \ldots, \bar{C}^n)}$, etc.

Observe that all observables of loop quantum gravity used here are self-adjoint operators over the boundary Hilbert space, and hence their eigenvalues are real. It therefore suffices to restrict integrations over the spectra of these operators to the real axes.

Now, in the saddle point approximation, the coefficients of terms linear in a, c^i vanish by definition of the saddle point. These imply that, at saddle point

$$\beta = \frac{S_A}{M_A}, \qquad P_i = M_{C^i}. \tag{13}$$

3.2. *Stability criteria*

Convergence of the integral (12) implies that the Hessian matrix (H) has to be positive definite, where

$$H = \begin{pmatrix} \beta M_{AA} - S_{AA} & \beta M_{AC^1} & \beta M_{AC^2} & \cdots & \beta M_{AC^n} \\ \beta M_{AC^1} & \beta M_{C^1C^1} & \beta M_{C^1C^2} & \cdots & \beta M_{C^1C^n} \\ \beta M_{AC^2} & \beta M_{C^2C^1} & \beta M_{C^2C^2} & \cdots & \beta M_{C^2C^n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \beta M_{AC^n} & \beta M_{C^nC^1} & \beta M_{C^nC^2} & \cdots & \beta M_{C^nC^n} \end{pmatrix}. \tag{14}$$

Here, all the derivatives are calculated at the saddle point. The Hessian matrix is real symmetric and hence it can be diagonalized by orthogonal matrix. So, positive definiteness of Hessian matrix boils down to the positivity of $(n+1)$ eigenvalues of Hessian matrix. Hence, the stability criteria is equivalent to the criteria of positivity of all eigenvalues of Hessian matrix and is given as

$$D_1 > 0, D_2 > 0, \ldots, D_{n+1} > 0, \tag{15}$$

where

$$D_1 = \beta M_{AA} - S_{AA}\,, \qquad D_2 = \begin{vmatrix} \beta M_{AA} - S_{AA} & \beta M_{AC^1} \\ \beta M_{AC^1} & \beta M_{C^1C^1} \end{vmatrix},$$

$$D_3 = \begin{vmatrix} \beta M_{AA} - S_{AA} & \beta M_{AC^1} & \beta M_{AC^2} \\ \beta M_{AC^1} & \beta M_{C^1C^1} & \beta M_{C^1C^2} \\ \beta M_{AC^2} & \beta M_{C^2C^1} & \beta M_{C^2C^2} \end{vmatrix}, \ldots, D_{n+1} = |H| \tag{16}$$

where $|H|$ = determinant of the Hessian matrix H.

Of course, (inverse) temperature β is assumed to be positive for a stable configuration. What is new is the requirement that temperature must increase with horizon area, inherent in the positivity of the quantity $(\beta M_{AA} - S_{AA})$ which appears in several of the stability criteria. If this is violated, as for example in case of the standard Schwarzschild black hole, thermal instability is inevitable.

The convexity property of the entropy follows from the condition of convergence of partition function under Gaussian fluctuations.[16–18] The thermal stability is related to the convexity property of entropy. Hence, the above conditions are correctly the conditions for thermal stability. For rotating charged horizons, Eqs. (15) and (16) reproduce the thermal stability criterion with $n = 2$, i.e. $D_1 > 0$, $D_2 > 0$, $D_3 > 0$ with the identification that charge of the black hole $(Q) = C^1$ and angular momentum of the black hole $(J) = C^2$. It can easily be checked that these three conditions correctly reproduce the earlier[12] seven conditions of thermal stability of charged rotating black holes. Equations (15) and (16) necessarily tell us that thermal stability of black hole is a consequence of the interplay among all the charges of the black hole.

As claimed in the Introduction, the thermal stability criteria above are derived by the application of standard statistical mechanical formalism to a quantum horizon characterized by various observables having discrete eigenvalue spectra. Thus, no aspect of classical geometry enters the derivation of these criteria. If the mass of the horizon is given in terms of its area and all the charges of the horizon, then it is possible, on the basis of our stability criteria, to predict which black holes will radiate away to extinction, and which ones might find some stability, and for what range of parameters. This is what is attempted in Sec. 4.

4. Predicting Thermal Stability of Black Holes

Note that in the stability criteria derived in Sec. 3, first- and second-order derivatives of the microcanonical entropy of the horizon at equilibrium play a crucial role, in making some of the criteria nontrivial. Thus, corrections to the microcanonical entropy beyond the Bekenstein–Hawking area law, arising due to quantum spacetime fluctuations, are very significant, because without these, some of the stability criteria might lose their essential physical content. It has been shown that[7] the microcanonical entropy for *macroscopic* isolated horizons has the form

$$S = S_{\rm BH} - \frac{3}{2}\log S_{\rm BH} + \mathcal{O}(S_{\rm BH}^{-1}), \tag{17}$$

$$S_{\rm BH} = \frac{A_h}{4A_P}, \qquad A_P \to \text{Planck area}. \tag{18}$$

In Ref. 7, the result (17) was derived for black holes in four-dimensional spacetime. This is based on a three-dimensional SU(2) Chern–Simons theory. Consideration of U(1) Gauge also gives same correction.[11] Since entropy is a physical quantity, it cannot depend on the choice of gauge fixing. We will assume similar correction of entropy for the examples that we will study, i.e. leading order entropy correction even in higher dimension due to quantum gravity being logarithmic with negative coefficient. Although it is an assumption, it can be argued heuristically as follows:[19] Gauss constraints restrict over the availability of phase space and hence degrees of freedom decreases. So, entropy decreases as a consequence of it. Now, without any constraint, entropy of black hole (S) is given as $S = \frac{A}{4A_P}$. So, leading order correction is expected to be logarithmic in area (A) with negative coefficient. $(4+1)$-dimensional Lorentz group, i.e. $SO(4,1)$ has ten generators. Among these, six generators correspond to rotation of four-dimensional space. So, analogy of $SO(3,1)$ implies $SU(2)\times SU(2)$ is the covering group of $SO(4)$ with six generators. Each of these six planes can be associated with an U(1) rotation. Hence, the coefficient of the $\log(A)$ term is expected to be double of $SO(3)$ case, i.e. $-(2\times 3/2) = -3$. Of course, this is heuristic, the exact number should be given by details of possible embeddings of SU(2) in the covering group of SO(4). The upshot is that correction is logarithmic in area with negative coefficient of order one.

4.1. *Uncharged rotating black hole in* $(4+1)$*-dimensional flat spacetime*

The properties of uncharged rotating black holes in $(N+1)$-dimensional spacetime have been studied in detail in Ref. 20. We extract out the necessary portions required for $(4+1)$-dimensional spacetime. The mass (M) of the black hole is given as

$$M = \frac{3\pi\mu}{8G}, \tag{19}$$

where μ is mass parameter of G which is the Newton constant for five-dimensional spacetime.

Since the spacetime is five-dimensional, there will be two directions of rotation for the black hole. The rotational parameters (a_1, a_2) are given as

$$a_1 = \frac{3J_1}{2M}, \qquad a_2 = \frac{3J_2}{2M}, \tag{20}$$

where J_1, J_2 are the two angular momentums of the black hole.

The horizon of the black hole (r_h) is given as

$$\Pi_{r_h} = \mu r_h^2 \,, \tag{21}$$

where

$$\Pi_r = (r^2 + a_1^2)(r^2 + a_2^2) \,. \tag{22}$$

Equations (21) and (22) together give

$$2r_h^2 = \mu - a_1^2 - a_2^2 + \sqrt{(\mu - a_1^2 - a_2^2)^2 - 4a_1^2 a_2^2} \,. \tag{23}$$

So, positivity of r_h^2 ensures that $\mu > (a_1^2 + a_2^2)$ and reality of r_h^2 ensures that either $\mu > (a_1 + a_2)^2$ or $\mu < (a_1 \sim a_2)^2$. These imply that reality and positivity of r_h^2 necessarily means $\mu > (a_1 + a_2)^2$. This mathematical inequality is the artifact of the fact that horizon of five-dimensional Myers–Perry black hole is formed only if mass dominates over rotation.

The surface gravity (κ) of black hole is given as

$$\kappa = \left. \frac{\frac{\partial(\Pi)}{\partial r} - 2\mu r}{2\mu r^2} \right|_{r_h} \,. \tag{24}$$

Equations (22)–(24) together give

$$\kappa = \frac{\sqrt{(\mu - a_1^2 - a_2^2)^2 - 4a_1^2 a_2^2}}{\mu r_h} \,. \tag{25}$$

The horizon area (A) of black hole is given as

$$A = \frac{16\pi GM \left(1 - \frac{a_1^2}{r_h^2 + a_1^2} - \frac{a_2^2}{r_h^2 + a_2^2}\right)}{3\kappa} \,. \tag{26}$$

Equation (26) implies that large horizon area (A) limit means large value of black hole mass (M) and small value of surface gravity (κ). Equation (25) implies that small value of κ means large value of μ and r_h.

Equations (23), (25) and (26) together give

$$A = \frac{16\pi GM}{3} \cdot \frac{1}{\sqrt{(\mu - a_1^2 - a_2^2)^2 - 4a_1^2 a_2^2}} \cdot \frac{r_h^4 - a_1^2 a_2^2}{r_h} \,. \tag{27}$$

Logarithm of Eq. (27) in the limit $\mu > (a_1 + a_2)^2$ gives

$$\ln(A) = \frac{3}{2}\ln(M) - \frac{27\pi}{32GM^3} \cdot (J_1^2 + J_2^2) + \mathcal{O}\left(\frac{J^4}{M^6}\right), \tag{28}$$

where we expressed μ, a_1, a_2 in terms of M, J_1, J_2 by relations (19) and (20). Here, $\mathcal{O}\big(\frac{J^4}{M^6}\big)$ is in terms of order $\frac{J_1^4}{M^6}$, $\frac{J_2^4}{M^6}$, $\frac{J_1^2 J_2^2}{M^6}$, etc. and we have thrown away irrelevant constants like $\ln(G)$, etc.

In large horizon area (A) limit, it can be easily shown that $D_1 = (\beta M_{AA} - S_{AA}) = -\frac{1}{6A^{4/3}A_P}$, in leading order. So, it is negative. This implies thermal instability of five-dimensional Myers–Perry black holes under Hawking radiation.

4.2. *Asymptotically ADS dyonic black holes with electric and magnetic charge*

The properties of asymptotically ADS dyonic black holes with electric and magnetic charge have been studied in detail in Ref. 21. The four-dimensional metric for this black hole is given as

$$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + R^2\,d\Omega^2\,, \tag{29}$$

where

$$\phi = \frac{\phi_3}{r^3} + \mathcal{O}(r^{-4})\,, \tag{30}$$

$$f = \frac{-\Lambda r^2}{3} + 1 - \frac{2M}{r} + \frac{Q^2+P^2}{r^2} + \frac{\Lambda\phi_3^2}{5r^4} + \mathcal{O}(r^{-5})\,, \tag{31}$$

$$R = r - \frac{3\phi_3^2}{20r^5} + \mathcal{O}(r^{-6})\,, \tag{32}$$

$$\phi_3 = \frac{g_0}{\Lambda}\int_{r_h}^{\infty}\frac{dr}{R^2}(Q^2\exp(2g_0\phi) - P^2\exp(-2g_0\phi))\,, \tag{33}$$

where r_h, Q and P are the radius of horizon, electric charge and magnetic charge of the black hole, respectively. $\Lambda(<0)$ is the cosmological constant, g_0 is diatonic coupling constant and ϕ is the diatonic field.

It is clear from Eq. (31) that unless diatonic field is too strong, its contribution on mass of black hole of large horizon is negligible. Again, Eqs. (30)–(33) together imply that weak diatonic field limit is possible if $Q^2, P^2 \ll A$.

So, area of black hole horizon (A) for weak field limit is given as $A = 4\pi R^2(r = r_h) \approx 4\pi r_h^2$ for large black hole.

The black hole horizon is given as a solution of $f(r = r_h) = 0$. Considering all the above equations, it turns out that

$$M \approx \frac{A^{3/2}}{48l^2\pi^{3/2}} + \frac{A^{1/2}}{4\pi^{1/2}} + \frac{\pi^{1/2}(Q^2+P^2)}{A^{1/2}}\,, \tag{34}$$

where $\Lambda = -1/l^2$, l is the cosmic length.

Retaining the leading terms in the horizon area, as before, Eqs. (13), (17) and (34) give the inverse temperature (β) as

$$\beta = \left(\frac{1}{A_P} - \frac{6}{A}\right)\Bigg/\left(\frac{A^{1/2}}{8l^2\pi^{3/2}} + \frac{1}{2\pi^{1/2}A^{1/2}} - \frac{2\pi^{1/2}(Q^2+P^2)}{A^{3/2}}\right). \tag{35}$$

Since we are dealing with macroscopic black holes with a large event horizon area $(A \gg A_P)$[12] and hence β is positive in weak field limit, i.e. $Q^2, P^2 \ll A$. One can also verify that unlike the asymptotically flat case, for anti-de Sitter black holes, the horizon temperature does increase with horizon area.

To complete the test for thermal stability, condition (15) has to be checked with $n = 2$. With the identification $C_1 = Q$ and $C_2 = P$, it can be easily shown from

Eqs. (15) and (16) that $D_1 = \left(\beta \cdot \left(\frac{1}{64l^2\pi^{3/2}A^{1/2}} - \frac{1}{16\pi^{1/2}A^{3/2}} + \frac{3\pi^{1/2}(Q^2+P^2)}{4A^{3/2}}\right) - \frac{3}{2A^2}\right)$ and is positive if $A \gg l^2$. In this limit ($A \gg A_P$, l^2, Q^2, P^2), it can easily be shown that D_2, D_3 are also positive. Thus, the asymptotically ADS dyonic large black holes with electric and magnetic charge turns out to be thermodynamically stable for a sufficiently large negative asymptotic curvature in weak field limit.

5. Summary

We reiterate that our analysis is quite independent of specific classical spacetime geometries, relying as it does on quantum aspects of spacetime. The construction of the partition function used standard formulations of equilibrium statistical mechanics augmented by results from canonical quantum gravity, with extra inputs regarding the behavior of the microcanonical entropy as a function of area *beyond the Bekenstein–Hawking area law*, as for instance derived from loop quantum gravity.[7] However, we emphasize that the results are more general than being restricted to any specific proposal for quantum spacetime geometry, requiring only certain functional dependences on horizon area and other parameters of statistical mechanical quantities like entropy. It also stands to reason that our stability criteria are useful for predicting the thermal behavior vis-a-vis Hawking radiation for specific astrophysical black holes.

It is also noteworthy that our approach is applicable to black holes with arbitrary "hairs" (charges) in Lorentzian spacetimes with arbitrary number of spatial dimensions. Black holes with arbitrary hairs had been studied in Refs. 22–28. In these references, it is more or less shown that energy of black hole in general depends on all its hairs as well. So, it is naturally expected that these hairs will govern thermal stability of black hole. In this sense, this paper covers the entire gamut of black hairs and their role on thermal stability of black hole.

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